# ARITHMETIC AND GEOMETRIC PROGRESSIONS

1) Complete the following sequences:

a) 3, 10, 17, ___, ___,..  b) 2, 6, 18, ___, ___,....

c) –11, -6, -1, ___, ___,...  d) –1 , 7, -49, ___, ___,..

2) Determine whether the following sequences are arithmetic or geometric progressions (or neither)

a) 5, -1, -7. –13,.....  b) 2, 3, 7, 8, 12, .....

c) 2, 1, $\frac{1}{2}$, $\frac{1}{4}$,.....  d) 27, 9, 3, 1,.....

3) 164 is the first term in an arithmetic progression, 173 is the second term in the sequence. Find the 93rd term in the sequence.

4) Find the $10^{th}$ of the progression: -3, 1, 5, 9, .....

5) Find the $7^{th}$ of the progression: 2, 4, 8, 16 .....

6) Find the sum of the first 100 terms of the progression: 1, 6, 11, 16,....

7) Find the sum of the first 50 terms of the progression: 1, 6, 36, 216,....

8) The first and the $6^{th}$ of a geometric progression are $\frac{5}{2}$ and $\frac{1215}{2}$. Find the value of r.

9) Find the sum to infinity of the geometric progression: 81, -27, 9,.....

10) The sum to infinity in a geometric progression is 200. Given that the first term is 52. Find the common ratio, r.

**SOLUTION**

1) Complete the following sequences:

a) 3, 10, 17, 24, 31,..  b) 2, 6, 18, 54, 162,....

c) –11, -6, -1, 4, 9,...  d) –1 , 7, -49, 343, -2401,..

2) Determine whether the following sequences are arithmetic or geometric progressions (or neither)

a) 5, -1, -7. –13,..... Arithmetic progression, difference +6

b) 2, 3, 7, 8, 12, ..... No progression

c) 2, 1, ½, ¼,..... Geometric progression, common ratio $\frac{1}{2}$

d) 27, 9, 3, 1,..... Geometric progression, common ratio $\frac{1}{3}$

3) 164 is the first term in an arithmetic progression, 173 is the second term in the sequence. Find the 93rd term in the sequence.

Difference $d = 173 - 164 = 9$

$a_n = a_1 + (n-1)d \rightarrow a_{93} = 164 + (93-1)9 = 164 + 92 \times 9 = 992$

The 93rd term in the sequence is 992

4) Find the 10th of the progression: -3, 1, 5, 9, .....

It is a AP (Arithmetic progression) with d = 4

$a_{10} = a_1 + (10-1)d \rightarrow a_{10} = -3 + 9 \times 4 = 33$

5) Find the 7th of the progression: 2, 4, 8, 16 .....

It is a GP (Geometric progression) with r = 2

$a_n = a_1 \times r^{n-1} \rightarrow a_7 = a_1 \times r^6 = 2 \times 2^6 = 2^7 = 128$

6) Find the sum of the first 100 terms of the progression: 1, 6, 11, 16,....

It is a AP (Arithmetic progression) with d = 5 and $a_1 = 1$

$S_n = \frac{(a_1 + a_n) \times n}{2}$ ; $a_{100} = 1 + 99 \times 5 = 496$

$S_{100} = \frac{(a_1 + a_{100}) \times 100}{2} = \frac{(1 + 496) \times 100}{2} = 24850$

7) Find the sum of the first 50 terms of the progression: 1, 6, 36, 216,....

It is a GP (Geometric progression) with r = 6 (>1) and $a_1 = 1$

$a_{50} = a_1 \times r^{49} = 1 \times 6^{49} = 6^{49}$

$S_n = \frac{a_n r - a_1}{r-1} \rightarrow S_{50} = \frac{a_{50} \times r - a_1}{r-1} = \frac{6^{49} \times 6 - 1}{6-1} = \frac{6^{50} - 1}{5} = 1.62 \times 10^{38}$

8) The first and the 6th of a geometric progression are $\frac{5}{2}$ and $\frac{1215}{2}$. Find the value of r.

$a_1 = \frac{5}{2}$ and $a_6 = \frac{1215}{2} \rightarrow a_6 = a_1 \times r^5 \Rightarrow \frac{1215}{2} = \frac{5}{2} \times r^5$

$r^5 = \frac{1215}{2} \div \frac{5}{2} = 243 \Rightarrow r = \sqrt[5]{243} = \sqrt[5]{3^5} \Rightarrow r = 3$

9) Find the sum to infinity of the geometric progression: 81, -27, 9,……

It is a GP with $r = -\frac{1}{3}$ $\quad S = \frac{a_1}{1-r} = \frac{81}{1+\frac{1}{3}} = \frac{81}{\frac{4}{3}} = \frac{81 \times 3}{4} = \frac{243}{4}$

10) The sum to infinity in a geometric progression is 200. Given that the first term is 52. Find the common ratio, r.

$S = \frac{a_1}{1-r} = 200 \rightarrow \frac{52}{1-r} = 200 \Rightarrow 52 = 200 \times (1-r) \Rightarrow 52 = 200 - 200r$

$52 - 200 = -200r \Rightarrow 200r = 148 \Rightarrow r = \frac{148}{200} = \frac{37}{50}$