ALGEBRAIC FRACTIONS 1

1. Simplify:

a) $\dfrac{x^4-2x^3-3x^2}{x^4-9x^2}$

b) $\dfrac{16x^4-1}{4x^2-4x+1}$

c) $\dfrac{x^5+6x^4+9x^3}{x^3+3x^2}$

d) $\dfrac{2x^5-32x}{6x^4+12x^3+24x^2+48x}$

e) $\dfrac{x^3-x}{x^3+3x^2+2x}$

f) $\dfrac{x^3-3x^2+3x-1}{x^3-2x^2+x}$

2) Work out and simplify:

a) $\left(\dfrac{3}{x}-\dfrac{2x}{x+1}\right)\cdot\dfrac{x^2+x}{x-1}$

b) $\dfrac{(x-1)^2}{2}\cdot\dfrac{1}{x^2-1}-\dfrac{3x}{(x+1)^2}$

SOLUTION

1. Simplify:

a) $$\frac{x^4-2x^3-3x^2}{x^4-9x^2}=\frac{x^2\left(x^2-2x-3\right)}{x^2\left(x^2-9\right)}=\frac{x^2(x-3)(x+1)}{x^2(x-3)(x+3)}=\frac{x+1}{x+3}$$

b) $$\frac{16x^4-1}{4x^2-4x+1}=\frac{\left(4x^2-1\right)\left(4x^2+1\right)}{(2x-1)^2}=\frac{(2x-1)(2x+1)\left(4x^2+1\right)}{(2x-1)^2}=$$
$$=\frac{(2x+1)\left(4x^2+1\right)}{(2x-1)}$$

c) $$\frac{x^5+6x^4+9x^3}{x^3+3x^2}=\frac{x^3\left(x^2+6x+9\right)}{x^2(x+3)}=\frac{x^3(x+3)^2}{x^2(x+3)}=x(x+3)=x^2+3x$$

d) $$\frac{2x^5-32x}{6x^4+12x^3+24x^2+48x}=\frac{2x\left(x^4-16\right)}{6x\left(x^3+2x^2+4x+8\right)}=$$

$$=\frac{2x\left(x^2-4\right)\left(x^2+4\right)}{6x\left(x^3+2x^2+4x+8\right)}=\frac{2x(x-2)(x+2)\left(x^2+4\right)}{6x(x+2)\left(x^2+4\right)}=\frac{x-2}{3}$$

e) $$\frac{x^3-x}{x^3+3x^2+2x}=\frac{x\left(x^2-1\right)}{x\left(x^2+3x+2\right)}=\frac{x(x-1)(x+1)}{x(x+1)(x+2)}=\frac{x-1}{x+2}$$

f) $$\frac{x^3-3x^2+3x-1}{x^3-2x^2+x}=\frac{(x-1)^3}{x(x-1)}=\frac{x-1}{x}$$

2) Work out and simplify:

a) $$\left(\frac{3}{x}-\frac{2x}{x+1}\right)\cdot\frac{x^2+x}{x-1}=\frac{3(x+1)-2x^2}{x(x+1)}\cdot\frac{x^2+x}{x-1}=$$
$$=\frac{-2x^2+3x+3}{x(x+1)}\cdot\frac{x(x+1)}{x-1}=\frac{-2x^2+3x+3}{x-1}$$

b) $$\frac{(x-1)^2}{2}\cdot\frac{1}{x^2-1}-\frac{3x}{(x+1)^2}=\frac{(x-1)(x-1)}{2(x+1)(x-1)}-\frac{3x}{(x+1)^2}=$$
$$=\frac{x-1}{2(x+1)}-\frac{3x}{(x+1)^2}=\frac{x^2-1-6x}{2(x+1)^2}=\frac{x^2-6x-1}{2(x+1)^2}$$