ALGEBRAIC FRACTIONS 2

Work out and simplify:

1.- $\frac{2}{x+2} - \frac{4}{x-4} - \frac{3x-6}{x^2-2x-8} =$

2.- $\left(1 - \frac{16}{x^2}\right) : \left(\frac{x^2-8x+16}{x}\right) =$

3.- $\frac{x^2-4}{x^2-5x+6} \cdot \frac{2x-6}{x^2-2x+1} \cdot \frac{3x^2-3}{6x+12} =$

4.- $\frac{x}{2x-4} - \frac{x^2-2}{x^2-4} + \frac{2}{x+2} =$

5.- $\left(1 + \frac{1}{x-1}\right) : \frac{x^2+x}{x^2-1} =$

6.- $\frac{x^2-9}{x^2-5x+6} \cdot \frac{x^2-4}{x^2-4x+4} \cdot \frac{x^2+x-6}{(x+3)^2} =$

SOLUTION

1.- $x^2-2x-8=(x+2)(x-4) \rightarrow$ LCF (denominators) $=(x+2)(x-4)$

$$\frac{2}{x+2}-\frac{4}{x-4}-\frac{3x-6}{x^2-2x-8}=$$

$$\frac{2(x-4)}{(x+2)(x-4)}-\frac{4(x+2)}{(x+2)(x-4)}-\frac{3x-6}{(x+2)(x-4)}=$$

$$=\frac{2x-8-(4x+8)-(3x-6)}{(x+2)(x-4)}=\frac{2x-8-4x-8-3x+6}{(x+2)(x-4)}=\frac{-5x+10}{x^2-2x-8}$$

2.- $\left(1-\frac{16}{x^2}\right):\left(\frac{x^2-8x+16}{x}\right)=\frac{x^2-16}{x^2}:\frac{x^2-8x+16}{x}=\frac{(x^2-16)\cdot x}{x^2(x^2-8x+16)}=$

Factorising polynomials (numerator and denominator):

$=\frac{x(x-4)(x+4)}{x^2(x-4)(x-4)}$ and simplifying: $\frac{x+4}{x(x-4)}=\frac{x+4}{x^2-4x}$

3.- $\frac{x^2-4}{x^2-5x+6}\cdot\frac{2x-6}{x^2-2x+1}\cdot\frac{3x^2-3}{6x+12}$ factorising all polynomials:

$$\frac{(x-2)(x+2)\cdot 2(x-3)\cdot 3(x+1)(x-1)}{(x-2)(x-3)\cdot(x-1)(x-1)\cdot 6(x+2)}=\frac{x+1}{x-1}$$

4.- LCF (denominators) $=2(x-2)(x+2)$

$$\frac{x}{2x-4}-\frac{x^2-2}{x^2-4}+\frac{2}{x+2}=\frac{x(x+2)}{2(x-2)(x+2)}-\frac{2(x^2-2)}{2(x-2)(x+2)}+$$

$$+\frac{4(x-2)}{2(x-2)(x+2)}=\frac{x^2+2x-(2x^2-4)+4x-8}{2(x-2)(x+2)}=$$

$$=\frac{x^2+2x-2x^2+4+4x-8}{2(x-2)(x+2)}=\frac{-x^2+6x-4}{2x^2-8}$$

5.- $\left(1+\frac{1}{x-1}\right):\frac{x^2+x}{x^2-1}=\left(\frac{x-1+1}{x-1}\right):\frac{x^2+x}{x^2-1}=\frac{x(x^2-1)}{(x-1)(x^2+x)}$

factorising: $\frac{x(x-1)(x+1)}{(x-1)\cdot x(x+1)}=1$

6.- $\frac{x^2-9}{x^2-5x+6}\cdot\frac{x^2-4}{x^2-4x+4}\cdot\frac{x^2+x-6}{(x+3)^2}=$ factorising all polynomials:

$$\frac{(x+3)(x-3)\cdot(x+2)(x-2)\cdot(x-2)(x+3)}{(x-2)(x-3)\cdot(x-2)(x-2)\cdot(x+3)(x+3)}=\frac{x+2}{x-2}$$