EQUATIONS 1

1. Solve the following biquadratic equations:

a) $9x^4 + 5x^2 - 4 = 0$

b) $(x^2 - 5)(x^2 - 3) = -1$

c) $x^4 - 61x^2 + 400 = 0$

d) $x^4 - 11x^2 + 28 = 0$

2. Solve the following rational equations:

a) $\frac{9}{x} - \frac{x}{3} = 2$

b) $\frac{4}{x} - \frac{2x+1}{5} = 1$

c) $\frac{1}{x} - \frac{9x+1}{x^2-2x} = \frac{-3x}{x-2}$

d) $\frac{x}{x-2} - \frac{2}{x-2} = \frac{6}{x}$

SOLUTION

1. a) $9x^4+5x^2-4=0 \rightarrow z=x^2 \Rightarrow 9z^2+5z-4=0 \Rightarrow z=\begin{cases}-1\\ 4/9\end{cases}$

then $x=\begin{cases}\pm\sqrt{-1}=?\\ \pm\sqrt{\frac{4}{9}}=\pm\frac{2}{3}\end{cases}$ Solution: $x=\frac{2}{3}; x=-\frac{2}{3}$

b) $(x^2-5)(x^2-3)=-1 \Rightarrow x^4-5x^2-3x^2+15=-1 \Rightarrow x^4-8x^2+16=0$
$\Rightarrow (x^2-4)^2=0 \Rightarrow x^2-4=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

c) $x^4-61x^2+400=0 \rightarrow z=x^2 \Rightarrow z^2-61z+400=0 \Rightarrow z=\frac{61\pm 11}{2}\begin{cases}36\\ 25\end{cases}$

then, solution: $x=\begin{cases}\pm\sqrt{36}=\pm 6\\ \pm\sqrt{25}=\pm 5\end{cases}$

d) $x^4-11x^2+28=0 \rightarrow z=x^2 \Rightarrow z^2-11z+28=0 \Rightarrow z=\frac{11\pm 3}{2}\begin{cases}7\\ 4\end{cases}$

then, solution: $x=\begin{cases}\pm\sqrt{7}\\ \pm\sqrt{4}=\pm 2\end{cases}$

2. a) $\frac{9}{x}-\frac{x}{3}=2 \Rightarrow \frac{27-x^2}{3x}=\frac{6x}{3x} \Rightarrow 27-x^2=6x \Rightarrow x^2+6x-27=0$

$x=\begin{cases}-9\\ 3\end{cases}$

b) $\frac{4}{x}-\frac{2x+1}{5}=1 \Rightarrow \frac{20-x(2x+1)}{5x}=\frac{5x}{5x}$

$\Rightarrow 20-2x^2-x=5x \Rightarrow -2x^2-6x+20=0$, simplifying:

$-x^2-3x+10=0 \Rightarrow x=\begin{cases}2\\ -5\end{cases}$

c) $\frac{1}{x}-\frac{9x+1}{x^2-2x}=\frac{-3x}{x-2}$ $LCF=x(x-2)$

$\frac{x-2}{x(x-2)}-\frac{9x+1}{x^2-2x}=\frac{-3x^2}{x(x-2)} \rightarrow x-2-(9x+1)=-3x^2$

$3x^2-8x-3=0 \rightarrow x=3; x=-\frac{1}{3}$

d) $\frac{x}{x-2}-\frac{2}{x-2}=\frac{6}{x}\Rightarrow\frac{x^2-2x}{x(x-2)}=\frac{6(x-2)}{x(x-2)}\Rightarrow x^2-8x+12=0\Rightarrow x=\begin{cases}6\\2\end{cases}$

Do you see that I'm going to have a problem with $x = 2$? This value would cause division by zero in the original equation! Since the solution is x = 6

g) $x^4-12x^2+36=\left(x^2+6\right)^2$

h) $100-9a^2=(10-3a)(10+3a)$

i) $25x^2y-y=y\left(25x^2-1\right)=y(5x-1)(5x+1)$

j) $16a^2b-9b=b\left(16a^2-9\right)=b(4a-3)(4a+3)$

k) $3a^3-27ab^4=3a\left(a^2-9b^4\right)=3a\left(a-3b^2\right)\left(a+3b^2\right)$