EQUATIONS 2

1. Solve the following irrational equations:

a) $x - \sqrt{3x - 5} = 3$

b) $x - \sqrt{3x - 2} = 0$

c) $2x - \sqrt{2x - 1} = 1$

2. Solve the following equations:

a) $\sqrt{2}(x^2 - 5x + 6)(x^3 - x) = 0$

b) $\frac{x}{8} - \frac{2}{81x^3} = 0$

c) $\frac{x+1}{x} - \frac{2}{x+1} = 1$

d) $x^3 - 8x^2 + 20x - 16 = 0$

SOLUTION

1. a) $x-\sqrt{3x-5}=3 \Rightarrow x-3=\sqrt{3x-5} \Rightarrow (x-3)^2=\left(\sqrt{3x-5}\right)^2$

$x^2-6x+9=3x-5 \Rightarrow x^2-9x+14=0 \rightarrow x=\dfrac{9\pm 5}{2}=\begin{cases}7\\2\end{cases}$

Check:

$7-\sqrt{3\cdot 7-5}=7-\sqrt{21-5}=7-\sqrt{16}=7-4=3 \Rightarrow x=7$ is a solution

$2-\sqrt{3\cdot 2-5}=2-\sqrt{6-5}=2-1=1\neq 3 \rightarrow x=2$ is NOT a solution

Answer: x = 7

b) $x-\sqrt{3x-2}=0 \Rightarrow x=\sqrt{3x-2} \Rightarrow x^2=\left(\sqrt{3x-2}\right)^2 \Rightarrow x^2=3x-2$

$\Rightarrow x^2-3x+2=0 \rightarrow x=\dfrac{3\pm 1}{2}=\begin{cases}2\\1\end{cases}$

Check: $1-\sqrt{3\cdot 1-2}=0 \Rightarrow 1-\sqrt{1}=0 \Rightarrow 0=0 \rightarrow x=1$ is a solution

$2-\sqrt{3\cdot 2-2}=0 \Rightarrow 2-\sqrt{4}=0 \Rightarrow 0=0 \rightarrow x=2$ is a solution

Answer: x = 1, x = 2

c) $2x-\sqrt{2x-1}=1 \Rightarrow 2x-1=\sqrt{2x-1} \Rightarrow (2x-1)^2=\left(\sqrt{2x-1}\right)^2$

$4x^2-4x+1=2x-1 \Rightarrow 4x^2-6x+2=0 \Rightarrow 2x^2-3x+1=0 \rightarrow x=\dfrac{3\pm 1}{4}=\begin{cases}1\\\dfrac{1}{2}\end{cases}$

Check:

$2\cdot 1-\sqrt{2\cdot 1-1}=2-\sqrt{1}=1 \rightarrow x=1$ is a solution

$2\cdot\dfrac{1}{2}-\sqrt{2\cdot\dfrac{1}{2}-1}=1-\sqrt{0}=1 \rightarrow x=\dfrac{1}{2}$ is a solution

Answer: x = 1, x = 1/2

2. a) $\sqrt{2}(x^2-5x+6)(x^3-x)=0 \Rightarrow \begin{cases}x^2-5x+6=0\\x^3-x=0\end{cases}$

$x^2-4x+6=0 \Rightarrow x=\dfrac{5\pm\sqrt{25-24}}{2}=\begin{cases}3\\2\end{cases}$

$x^3-x=0 \Rightarrow x(x^2-1)=0 \Rightarrow \begin{cases}x=0\\x^2-1=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1\end{cases}$

Answer: $\underline{x=3}$, $\underline{x=2}$, $\underline{x=0}$, $\underline{x=1}$ y $\underline{x=-1}$

b) $\dfrac{x}{8}-\dfrac{2}{81x^3}=0$ LCF= $8\cdot 81x^3=648x^3$

$$\frac{81x^4}{648x^3}-\frac{16}{648x^3}=0\Rightarrow 81x^4-16=0$$

$$\Rightarrow 81x^4=16\Rightarrow x^4=\frac{81}{16}\Rightarrow x=\pm\sqrt[4]{\frac{81}{16}}=\pm\frac{3}{2}$$ Answer: $\underline{x=3/2}$ y $\underline{x=-3/2}$

c) $\frac{x+1}{x}-\frac{2}{x+1}=1\Rightarrow \frac{(x+1)(x+1)}{x(x+1)}-\frac{2x}{x(x+1)}=\frac{x(x+1)}{x(x+1)}$

$\Rightarrow x^2+2x+1-2x=x^2+x\Rightarrow 1=x$ Answer: x = 1

d) $x^3-8x^2+20x-16=0$, factorising:

Factors of 16: $\pm1,\pm2,\pm4,\pm8,\pm16$, cheking, we see $P(2)=8-32+40-16=0$

	1	-8	+20	-16
2		+2	-12	+16
	1	-6	+8	
				0

$x^3-8x^2+20x-16=(x-2)(x^2-6x+8)$

equation:

$(x-2)(x^2-6x+8)=0$

So: $\begin{cases} x-2=0\Rightarrow x=2 \\ x^2-6x+8=0\Rightarrow x=2,x=4 \end{cases}$ Answer **x = 2** y **x = 4**