FUNCTIONS 2

1. Find the vertex and the intercepts of the quadratic function: **f(x)=-x²+7x**
Sketch the graph.

2. A small plane flies between Cádiz and Ceuta. Its flight height is determined by the rule: $h(x) = -30x^2 + 900x$, where h(x) is the height of the plane in metres at x minutes after take-off from Cádiz. Draw the graph and work out:
a) The maximum height the plane reaches.
b) The duration of the flight.

3. The teachers in the school decide to play the lottery together. If they win, the prize is $600 000.
a) Complete the table that shows how much each teacher will get, depending on how many of them contribute to pay the tickets.

Number of teachers	1	2	3	4		
Amount each will receive	600000	300000			120000	60000

b) Describe a rule that fits the table.
c) Graph the function that shows how much each teacher will receive.

4. The labels stuck on tins of tomatoes are rectangular which an area of 18 cm². What are the possible lengths of the base and height? Make a table with some of the possibilities and graph the function. Find the rule. What type of function is it?

5. Margaret has a room, given to her by her parents, to meet up with her friends. In one of the meetings they decide to turn it into a cinema club, the equipment for which would cost 20 000 euros.
a) If the cost are going to be shared equally between all the future members, how many are needed if each member contributes 400 euros? And what if they decide to contribute 500 euros?
b) Analyse and describe the type of relation between the number of members and the investment needed to buy the equipment.
c) Graph the function.

6. Sketch the graph of a function using the following information:
a) A local maximum in (3,3)
b) Vertical asymptote x = 0 and horizontal asymptote y=1
c) Increasing in $(-\infty,0)$

7. Solve by substitution and graphically the simultaneous equations: $\left.\begin{array}{l} y = -x^2 + 4 \\ y = -3x \end{array}\right\}$
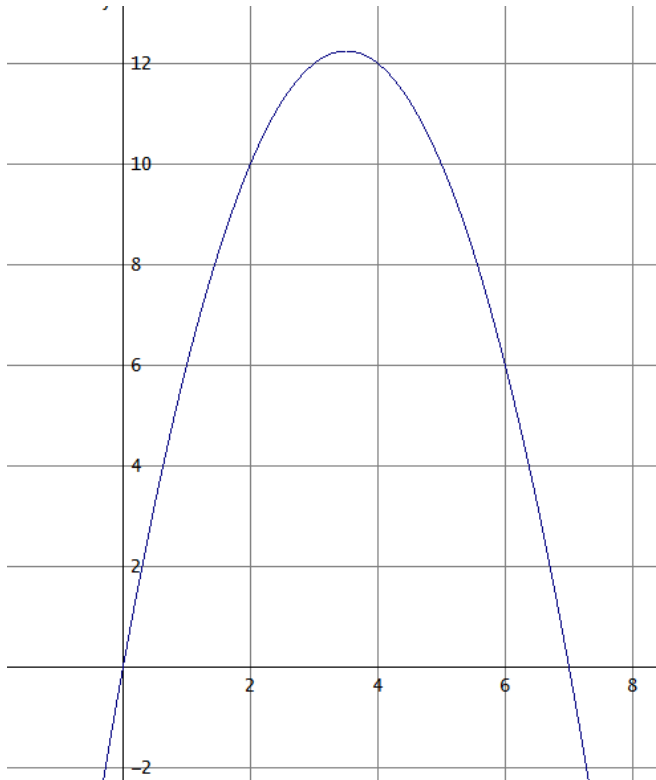
SOLUTION

1. Find the vertex and the intercepts of the quadratic function: $f(x) = -x^2 + 7x$
Sketch the graph.

Vertex $x = \dfrac{-7}{-2} = \dfrac{7}{2} \rightarrow y = -\left(\dfrac{7}{2}\right)^2 + 7 \cdot \dfrac{7}{2} = -\dfrac{49}{4} + \dfrac{49}{2} = \dfrac{49}{4} \rightarrow V\left(\dfrac{7}{2}, \dfrac{49}{4}\right)$

Intercepts: $x = 0 \rightarrow y = 0$

$$y = 0 \rightarrow -x^2 + 7x = 0 \rightarrow x(-x + 7) = 0 \rightarrow \begin{cases} x = 0 \\ x = 7 \end{cases}$$

2. A small plane flies between Cádiz and Ceuta. Its flight height is determined by the rule: $h(x) = -30x^2 + 900x$

Where h(x) is the height of the plane in metres at x minutes after take-off from Cádiz. Draw the graph and work out:
    a) The maximum height the plane reaches.
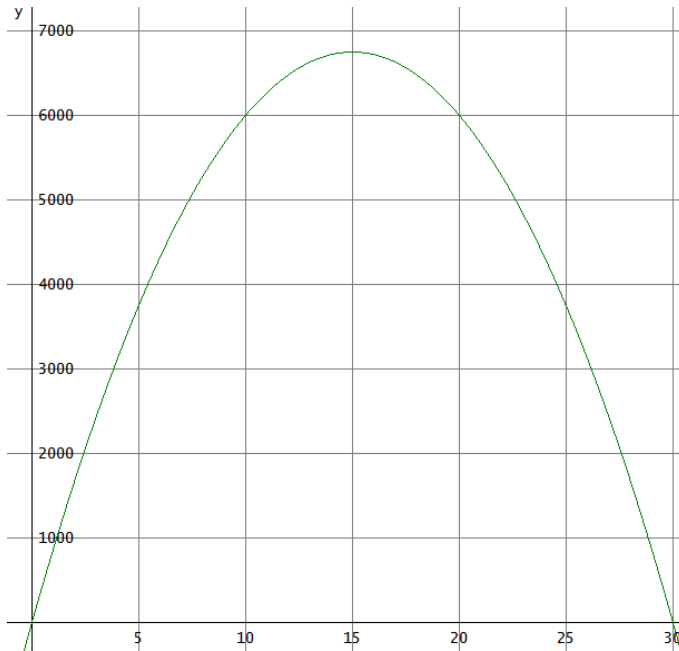   b) The duration of the flight.
It is a quadratic function:

Vertex $x = \dfrac{-900}{-60} = 15 \rightarrow y = -30 \cdot 15^2 + 900 \cdot 15 = 6750 \rightarrow V(15{,}6750)$

Maximum $\rightarrow (15{,}6750)$

Intercepts: $x = 0 \rightarrow y = 0$

$$y = 0 \rightarrow -30x^2 + 900x = 0 \rightarrow x(-30x + 900) = 0 \rightarrow \begin{cases} x = 0 \\ x = 30 \end{cases}$$

duration of the flight 30 minutes

3. The teachers in the school decide to play the lottery together. If they win, the prize is $600 000.
a) Complete the table that shows how much each teacher will get, depending on how many of them contribute to pay the tickets.

Number of teachers	1	2	3	4	5	10
Amount each will receive	600000	300000	200000	150000	120000	60000

b) Describe a rule that fits the table. $y = \dfrac{600000}{x}$

c) Graph the function that shows how much each teacher will receive.
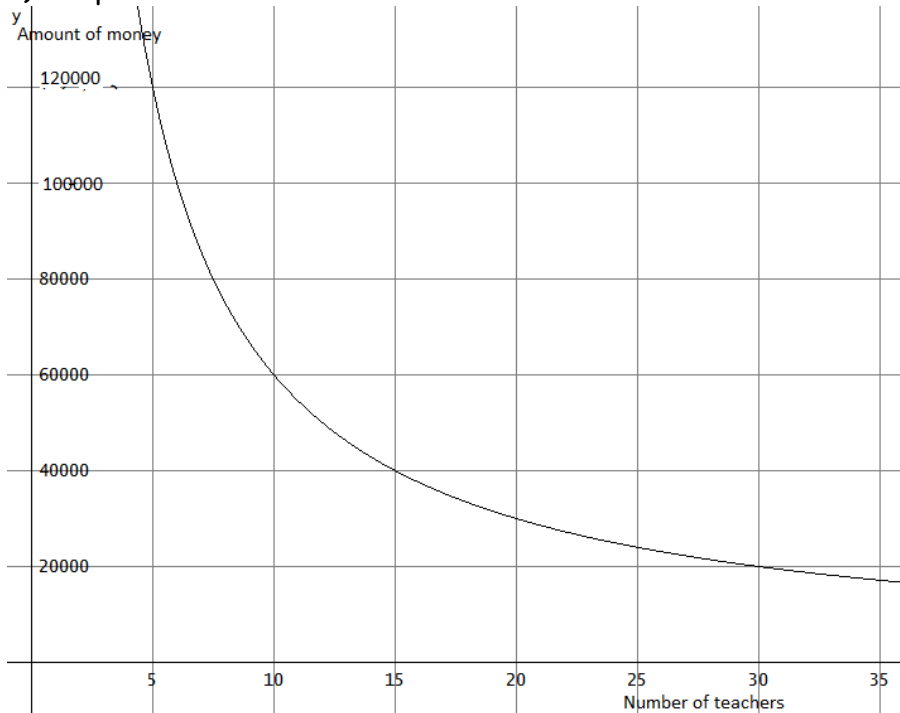
4. The labels stuck on tins of tomatoes are rectangular which an area of 18 cm^2. What are the possible lengths of the base and height? Make a table with some of the possibilities and graph the function. Find the rule. What type of function is it?

Length of the base	1	2	3	6	9	18
Height	18	9	6	3	2	1

Rule: $y = \dfrac{18}{x}$ It is an inverse proportion function, a hyperbola
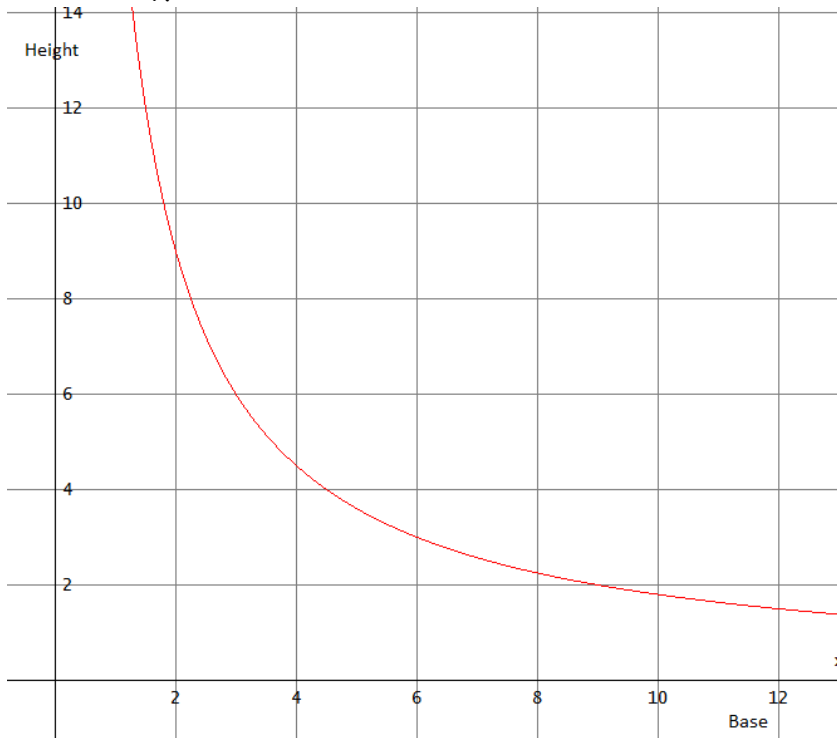
5. Margaret has a room, given to her by her parents, to meet up with her friends. In one of the meetings they decide to turn it into a cinema club, the equipment for which would cost 20 000 euros.

   a) If the cost are going to be shared equally between all the future members, how many are needed if each member contributes 400 euros? And what if they decide to contribute 500 euros?

   20000 : 400 = 50 members
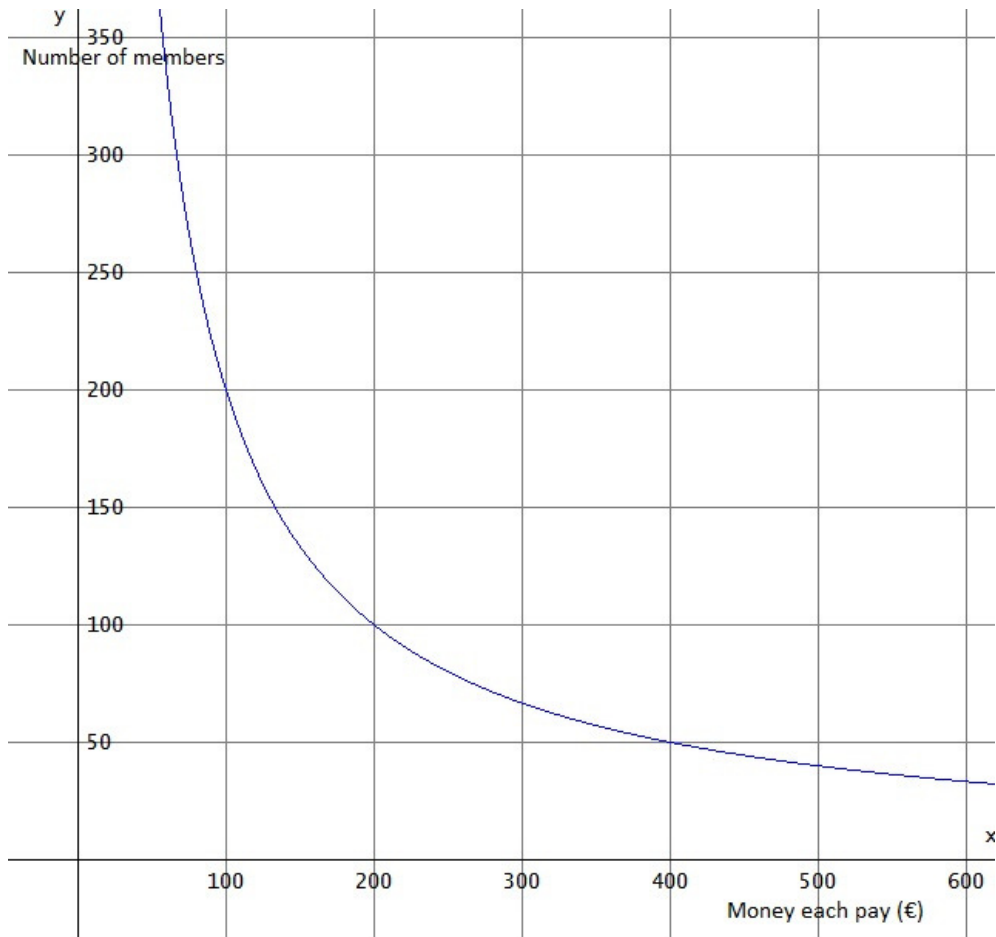
   20000 : 500 = 40 members

   b) Analyse and describe the type of relation between the number of members and the investment needed to buy the equipment.

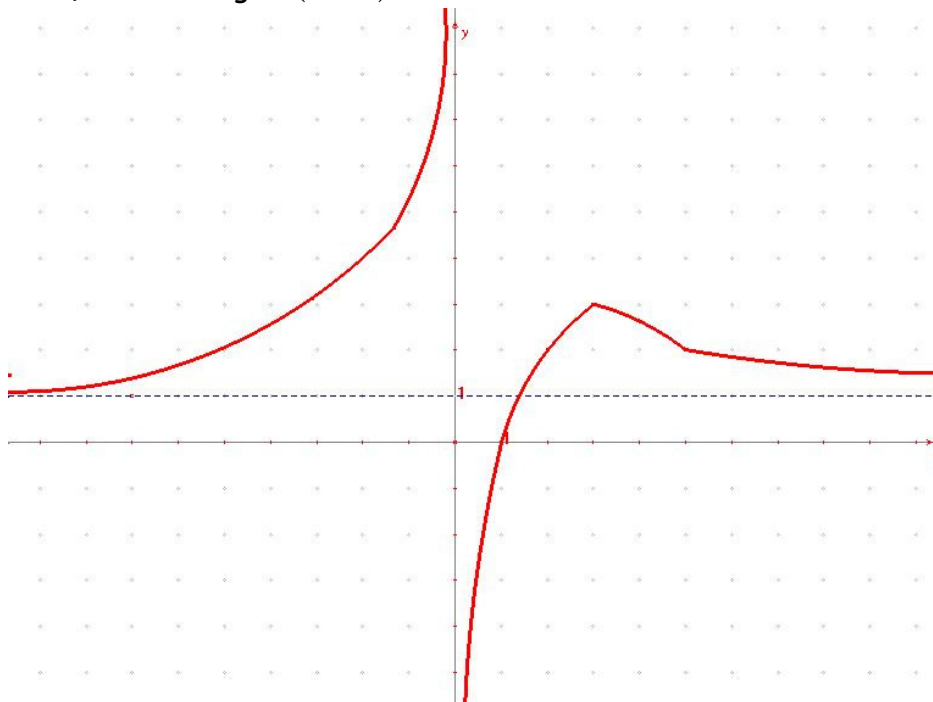   More money less members, inverse proportion function

   It is a hyperbola

   The rule is $y = \dfrac{20000}{x}$

   c) Graph the function.

6. Sketch the graph of a function using the following information:
   a) A local maximum in (3,3)
   b) Vertical asymptote x = 0 and horizontal asymptote y=1
   c) Increasing in $(-\infty,0)$

7. Solve by substitution and graphically the simultaneous equations: $\left.\begin{array}{l} y = -x^2 + 4 \\ y = -3x \end{array}\right\}$

$\left.\begin{array}{l} y = -x^2 + 4 \\ y = -3x \end{array}\right\} \rightarrow -x^2 + 4 = -3x$

$x^2 - 3x - 4 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 16}}{2} = \left\{\begin{array}{l} 4 \\ -1 \end{array}\right.$

$\left\{\begin{array}{l} x_1 = 4 \rightarrow y = -3 \cdot 4 = -12 \\ x_2 = -1 \rightarrow y = -3 \cdot (-1) = 3 \end{array}\right.$

SOLUTION

$\left\{\begin{array}{ll} x_1 = 4 & y_1 = -12 \\ x_2 = -1 & y_2 = 3 \end{array}\right.$

$y = -x^2 + 4$

Parabola with vertex (0,4)
the line of symmetry is   x = 0

It is open downward (a<0)
x and y intercepts: (0,4)
$0 = -x^2 + 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$y = -3x$ It is a line passing through the point
(0,0) and slope – 3

The graph intercepts in points
(4, -12) y (-1, 3) SOLUTION