POLYNOMIALS 2

1. Factorise the following polynomials:

a) $x^4 - x^3 - 9x^2 + 9x$

b) $2x^3 + 5x^2 + x - 2$

c) $x^4 + 2x^3 - 3x^2 - 4x + 4$

d) $2x^4 - 5x^3 + 5x - 2$

e) $x^3 + 4x^2 + 5x + 2$

f) $x^5 - 8x^4 + 11x^3 + 32x^2 - 60x$

2. Find the roots which are integers:

a) $x^4 + 3x^3 - 7x^2 - 27x - 18$

b) $5x^3 - 20x^2 - 20x + 80$

c) $3x^4 - 2x^2 + 6x + 1$

d) $x^4 - 6x^3 + 7x^2 + 6x - 8$

SOLUTION

1. Factorise the following polynomials:

a) $x^4 - x^3 - 9x^2 + 9x$

$x^4 - x^3 - 9x^2 + 9x = x(x^3 - x^2 - 9x + 9)$ Factors(9): $\pm 1, \pm 3, \pm 9$

$P(1) = 1^3 - 1^2 - 9 \cdot 1 + 9 = 0;$

	1	-1	-9	+9
1		1	0	-9
	1	0	-9	0

$x^4 - x^3 - 9x^2 + 9x = x(x^3 - x^2 - 9x + 9) =$
$= x(x-1)(x^2 - 9) = x(x-1)(x-3)(x+3)$

b) $2x^3 + 5x^2 + x - 2$ Factors(2): $\pm 1, \pm 2$

$P(1) = 2 \cdot 1^3 + 5 \cdot 1^2 + 1 - 2 \neq 0;\ P(-1) = 2 \cdot (-1)^3 + 5 \cdot (-1)^2 - 1 - 2 = 0$

	2	+5	+1	-2
-1		-2	-3	+2
	2	+3	-2	0

$2x^2 + 3x - 2 = 0; x = \dfrac{-3 \pm \sqrt{25}}{4} = \begin{cases} \dfrac{1}{2} \\ -2 \end{cases}$

$2x^3 + 5x^2 + x - 2 = 2(x+1)(x+2)\left(x - \dfrac{1}{2}\right)$

c) $x^4 + 2x^3 - 3x^2 - 4x + 4$ (Sol: $(x-1)^2(x+2)^2$)

d) $2x^4 - 5x^3 + 5x - 2$ (Sol: $(x-1)(x+1)(x-2)(2x-1)$)

e) $x^3 + 4x^2 + 5x + 2$ (Sol: $(x+2)(x+1)^2$)

f) $x^5 - 8x^4 + 11x^3 + 32x^2 - 60x$ (Sol: $x(x-2)(x+2)(x-3)(x-5)$)

2. Find the roots which are integers:

a) $x^4 + 3x^3 - 7x^2 - 27x - 18$ Factors(18): $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$P(1) = 1^4 + 3 \cdot 1^3 - 7 \cdot 1^2 - 27 - 18 \neq 0;\ P(-1) = 1 - 3 - 7 + 27 - 18 = 0;$

$P(2) = 2^4 + 3 \cdot 2^3 - 7 \cdot 2^2 - 27 \cdot 2 - 18 = 16 + 24 - 28 + 54 - 18 \neq 0$

$P(-2) = (-2)^4 + 3 \cdot (-2)^3 - 7 \cdot (-2)^2 + 54 - 18 = 16 - 24 - 28 + 54 - 18 = 0$

	1	+3	-7	-27	-18
-1		-1	-2	+9	+18
	1	+2	-9	-18	0
-2		-2	0	+18	
	1	0	-9	0	

$x^2 - 9 = 0 \rightarrow x = \pm 3$

Roots: -1, -2, +3, -3

All integers

b) $5x^3 - 20x^2 - 20x + 80$ (Sol: 2, -2, 4)

c) $3x^4 - 2x^2 + 6x + 1$ (Sol: there is no solution)

d) $x^4 - 6x^3 + 7x^2 + 6x - 8$ (Sol: 1, -1, 2 y 4)