SIMULTANEOUS EQUATIONS 2

Solve the following simultaneous equations:

1) $\left.\begin{array}{l} y = x^2 - 3x \\ y - 2x + 6 = 0 \end{array}\right\}$

2) $\left.\begin{array}{l} \frac{2}{x} + \frac{3}{y} = 3 \\ x + y = 4 \end{array}\right\}$

3) $\left.\begin{array}{l} 2x + y = 6 \\ \sqrt{x} - y = -3 \end{array}\right\}$

4) $\left.\begin{array}{l} \frac{3}{x} - \frac{x}{y} = 0 \\ 2x - y = 3 \end{array}\right\}$

5) $\left.\begin{array}{l} y = x^2 + 3x \\ y - 4x - 2 = 0 \end{array}\right\}$

6) $\left.\begin{array}{l} \frac{1}{x+y} = \frac{2}{5} \\ \frac{1}{x} + \frac{1}{y} = \frac{5}{2} \end{array}\right\}$

SOLUTION

1) $\left.\begin{matrix} y = x^2 - 3x \\ y - 2x + 6 = 0 \end{matrix}\right\} \rightarrow \left.\begin{matrix} y = x^2 - 3x \\ y = 2x - 6 \end{matrix}\right\} \Rightarrow x^2 - 3x = 2x - 6 \Rightarrow x^2 - 5x + 6 = 0$

$x = \begin{cases} 3 \rightarrow y = 2 \cdot 3 - 6 = 6 - 6 = 0 \\ 2 \rightarrow y = 2 \cdot 2 - 6 = 4 - 6 = -2 \end{cases}$ CONSISTENT INDEPENDENT SYSTEM

SOLUTION: $\begin{cases} x = 3, y = 0 \\ x = 2, y = -2 \end{cases}$

2) $\left.\begin{matrix} \frac{2}{x} + \frac{3}{y} = 3 \\ x + y = 4 \end{matrix}\right\} \rightarrow \left.\begin{matrix} 2y + 3x = 3xy \\ x + y = 4 \end{matrix}\right\} \rightarrow y = 4 - x \Rightarrow 2(4 - x) + 3x = 3x(4 - x)$

$3x^2 - 11x + 8 = 0 \rightarrow x = \frac{11 \pm \sqrt{121 - 96}}{6} = \frac{11 \pm 5}{6} = \begin{cases} \frac{8}{3} \rightarrow y = 4 - \frac{8}{3} = \frac{4}{3} \\ 1 \rightarrow y = 4 - 1 = 3 \end{cases}$

CONSISTENT INDEPENDENT SYSTEM SOLUTION: $\begin{cases} x = \frac{8}{3}, y = \frac{4}{3} \\ x = 1, y = 3 \end{cases}$

3) $\left.\begin{matrix} 2x + y = 6 \\ \sqrt{x} - y = -3 \end{matrix}\right\} \rightarrow \left.\begin{matrix} y = 6 - 2x \\ y = \sqrt{x} + 3 \end{matrix}\right\} \Rightarrow 6 - 2x = \sqrt{x} + 3 \Rightarrow 3 - 2x = \sqrt{x}$

$(3 - 2x)^2 = \left(\sqrt{x}\right)^2 \Rightarrow 9 - 12x + 4x^2 = x \Rightarrow 4x^2 - 13x + 9 = 0 \rightarrow x = \begin{cases} \frac{9}{4} \\ 1 \end{cases}$

Check: $3 - 2x = \sqrt{x} \rightarrow \begin{cases} 3 - 2 \cdot \frac{9}{4} = \sqrt{\frac{9}{4}} \rightarrow 3 - \frac{9}{2} \neq \frac{3}{2} \quad \text{No} \\ 3 - 2 \cdot 1 = \sqrt{1} \rightarrow 3 - 2 = 1 \quad \text{Yes} \end{cases}$

CONSISTENT INDEPENDENT SYSTEM SOLUTION: $x = 1, y = 4$

4) $\left.\begin{matrix} \frac{3}{x} - \frac{x}{y} = 0 \\ 2x - y = 3 \end{matrix}\right\} \rightarrow \left.\begin{matrix} 3y - x^2 = 0 \\ 2x - y = 3 \end{matrix}\right\} \rightarrow y = 2x - 3 \Rightarrow 3(2x - 3) - x^2 = 0$

$x^2 - 6x + 9 = 0 \rightarrow x = 3 \Rightarrow y = 6 - 3 = 3$

CONSISTENT INDEPENDENT SYSTEM SOLUTION: $x = 3, y = 3$

5) $\left.\begin{array}{l} y = x^2 + 3x \\ y - 4x - 2 = 0 \end{array}\right\} \rightarrow \left.\begin{array}{l} y = x^2 + 3x \\ y = 4x + 2 \end{array}\right\} \Rightarrow x^2 + 3x = 4x + 2 \Rightarrow x^2 - x - 2 = 0$

$$x = \begin{cases} 2 \\ -1 \end{cases} \rightarrow y = \begin{cases} 4 \cdot 2 + 2 = 10 \\ 4 \cdot (-1) + 2 = -2 \end{cases}$$

CONSISTENT INDEPENDENT SYSTEM SOLUTION: $\begin{cases} x = 2,\ y = 10 \\ x = -1,\ y = -2 \end{cases}$

6) $\left.\begin{array}{l} \dfrac{1}{x+y} = \dfrac{2}{5} \\ \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{5}{2} \end{array}\right\} \rightarrow \left.\begin{array}{l} 5 = 2x + 2y \\ 2y + 2x = 5xy \end{array}\right\} \rightarrow \left.\begin{array}{l} 2x + 2y = 5 \\ 2x + 2y = 5xy \end{array}\right\} \Rightarrow 5xy = 5 \Rightarrow xy = 1 \Rightarrow y = \dfrac{1}{x}$

$$2x + 2y = 5 \Rightarrow 2x + \frac{2}{x} = 5 \Rightarrow 2x^2 + 2 = 5x \Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow x = \begin{cases} 2 \\ \dfrac{1}{2} \end{cases}$$

$$y = \frac{1}{x} = \begin{cases} \dfrac{1}{2} \\ 2 \end{cases}$$

CONSISTENT INDEPENDENT SYSTEM SOLUTION: $\begin{cases} x = 2,\ y = \dfrac{1}{2} \\ x = \dfrac{1}{2},\ y = 2 \end{cases}$