TRIGONOMETRY 3

1. Suppose that $\cos\alpha = -\frac{3}{5}$ and α lies in quadrant II. Find the other trigonometric ratios for α.

2. Suppose that $\sin\alpha = \frac{\sqrt{2}}{5}$ and α lies in quadrant I. Find the other trigonometric ratios for α.

3. Suppose that $\tan\alpha = \frac{2}{3}$ and α lies in quadrant III. Find the other trigonometric ratios for α.

4. Find the exact value of sin 315°.

5. Find the exact value of tan 210°.

6. Find the exact value of cos 135°.

7. Two men on opposite sides of a TV tower of height 24 m notice the angle of elevation of the top of this tower to be 45° and 60° respectively. Find the distance between the two men.

8. Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 30° and 60° respectively. If the height of the building is known to be h =50 m, find the distance between the two men.

9. A rectangle has sides of length 12 cm and x cm. The acute angle between the diagonal of the rectangle is 40°. Determine x.

10. A 5.2 m ladder leans against a wall. The bottom of the ladder is 1.9 m from the wall. What angle does the ladder make with the ground ?

SOLUTION

1. Suppose that $\cos\alpha = -\frac{3}{5}$ and α lies in quadrant II. Find the other trigonometric ratios for α.

In quadrant II, $\sin\alpha > 0$, $\cos\alpha < 0$, $\tan\alpha < 0$

$$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \sin^2\alpha = 1 - \cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25} \rightarrow \sin\alpha = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$$

2. Suppose that $\sin\alpha = \frac{\sqrt{2}}{5}$ and α lies in quadrant I. Find the other trigonometric ratios for α.

In quadrant I, $\sin\alpha > 0$, $\cos\alpha > 0$, $\tan\alpha > 0$

$$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{2}{25} = \frac{23}{25} \rightarrow \cos\alpha = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{2}}{5}}{\frac{\sqrt{23}}{5}} = \frac{\sqrt{2}}{\sqrt{23}} = \frac{\sqrt{46}}{23}$$

3. Suppose that $\tan\alpha = \frac{2}{3}$ and α lies in quadrant III. Find the other trigonometric ratios for α.

In quadrant III, $\sin\alpha < 0$, $\cos\alpha < 0$, $\tan\alpha > 0$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1 + \frac{4}{9} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{9}{13} \rightarrow \cos\alpha = -\sqrt{\frac{9}{13}} = -\frac{3\sqrt{13}}{13}$$

$$\sin^2\alpha + \cos = 1 \Rightarrow \sin^2\alpha = 1 - \frac{9}{13} = \frac{4}{13} \rightarrow \sin\alpha = -\sqrt{\frac{4}{13}} = -\frac{2\sqrt{13}}{13}$$

4. Find the exact value of sin 315°.

The terminal side of the angle 315° is in the 4th quadrant, so 360°-315°=45°

45° is the reference angle, also sin is negative in the 4th

Thus, $\sin 315^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$

5. Find the exact value of tan 210°.

The terminal side of the angle 210° is in the 3rd quadrant, so 210°-180°=30°

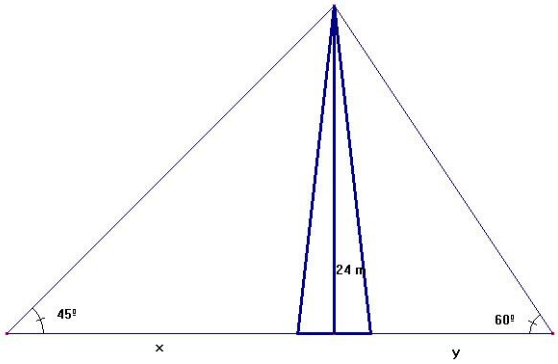
30° is the reference angle, also tan is positive in the 3rd

Thus, $\tan 210^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}$

6. Find the exact value of cos 135°.
The terminal side of the angle 135° is in the 2nd quadrant, so 180°-135°=45°
45° is the reference angle, also cos is negative in the 2nd

Thus, $cos\,135^\circ = -cos\,45^\circ = -\frac{\sqrt{2}}{2}$

7. Two men on opposite sides of a TV tower of height 24 m notice the angle of elevation of the top of this tower to be 45° and 60° respectively. Find the distance between the two men.

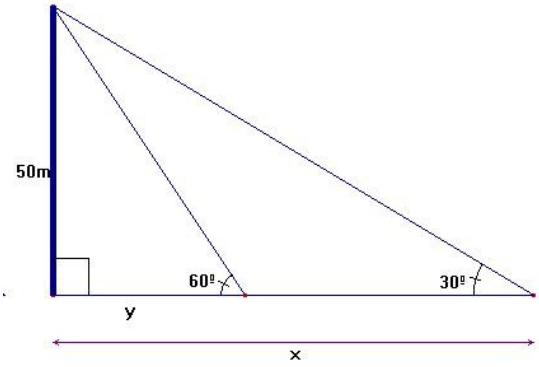


$$tan\,45^\circ = \frac{24}{x} \rightarrow x = \frac{24}{1} = 24m$$

$$tan\,60^\circ = \frac{24}{y} \rightarrow y = \frac{24}{\sqrt{3}} = 13.86m$$

Distance between the two men: 24 + 13.86 = 37.86 metres

8. Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 30° and 60° respectively. If the height of the building is known to be h =50 m, find the distance between the two men.

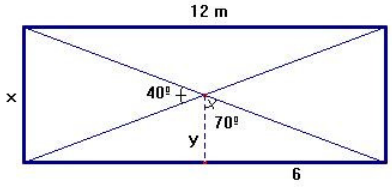


$$tan\,60^\circ = \frac{50}{y} \rightarrow y = \frac{50}{\sqrt{3}} = 28.87m$$

$$tan\,30^\circ = \frac{50}{x} \rightarrow x = \frac{50}{\sqrt{3}/3} = 86.6m$$

Distance between the two men: 86.6 – 28.87 = 57.73 metres

9. A rectangle has sides of length 12 cm and x cm. The acute angle between the diagonals of the rectangle is 40°. Determine x.

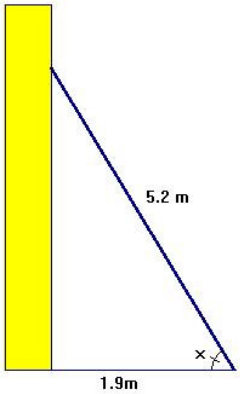


180° - 40° = 140°; 140° : 2 = 70°

$$tan\,70^\circ = \frac{6}{y} \rightarrow y = \frac{6}{\sqrt{3}} = 3.464$$

$$x = 2 \times 3.464 = 6.93\ m$$

10. A 5.2 m ladder leans against a wall. The bottom of the ladder is 1.9 m from the wall. What angle does the ladder make with the ground?



$$cos\,x = \frac{1.9}{5.2} \rightarrow x = arccos\left(\frac{1.9}{5.2}\right) = 68^\circ 34'$$