WORD PROBLEMS 2

1) The width of a rectangle is 5 metres less than its length. The area is 84 square metres. Find the dimensions of the rectangle.

2) The product of two consecutive odd integers is 1 less than twice their sum. Find the two integers.

3) One leg of a right triangle is one centimetre shorter than the other leg. If the hypotenuse is 5 cm, find the length of the shorter leg.

4) The product of two consecutive negative integers is 1260. What are the numbers?

5) A garden measuring 12 metres by 16 metres is to have a pedestrian pathway installed all around it, increasing the total area to 285 square metres. What will be the width of the pathway?

6) Find the fraction which becomes $\frac{1}{2}$ when the denominator is increased by 5 and is equal to $\frac{1}{3}$ when the numerator is diminished by 4.

7) A lady has 50 cents and 2€ coins in her purse. She has 90 coins in all and their total value is 105€. How many 2€ coins does she have?

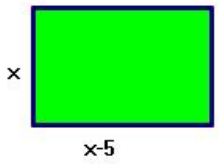
8) A mixture containing 6% boric acid is to be mixed with 4 litres of a mixture that is 15% acid in order to obtain a solution that is 12% acid. How much of the 6% solution must be used?

9) The owner of a candy store mixes two types of candies. She decides to create a 20-pound mixture of raspberry-flavored gumdrops and cherry-flavored jelly beans. The gumdrops sell for \$0.95 per pound and the jelly beans sell for \$1.20 per pound. She plans to sell the mix for \$1.10 per pound. How many pounds of each candy should she use in her mix?

10) A father is 28 years older than his daughter. In 2 years' time he will be 3 times as old as his daughter. Find their present ages.

ANSWERS

1. The width of a rectangle is 5 metres less than its length. The area is 84 square metres. Find the dimensions of the rectangle.



Length x, width x – 5

Área

$$A = x(x-5) = 84 \rightarrow x(x-5) = 84 \rightarrow x^2 - 5x - 84 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 4 \cdot 84}}{2} = \frac{5 \pm 19}{2} = \begin{cases} 12 \\ -7 \end{cases}$$

Answer: Length 12 metres and width 7 metres

2. The product of two consecutive odd integers is 1 less than twice their sum. Find the two integers.

Consecutive odd numbers: First: $2x+1$ Second: $2x+3$

Equation: $(2x+1)(2x+3) = 2(2x+1+2x+3) - 1$

$$4x^2 + 2x + 6x + 3 = 2(4x+4) - 1 \rightarrow 4x^2 + 8x + 3 = 8x + 8 - 1$$

$$4x^2 - 4 = 0 \rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Answer: First number $2+1=3$ or $-2+1=-1$

Second number $2+3=5$ or $-2+3=1$

Solution: Numbers are 3 and 5 or -1 and 1

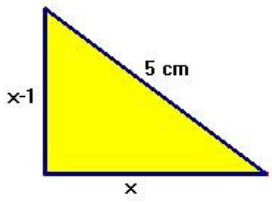
3. One leg of a right triangle is one centimetre shorter than the other leg. If the hypotenuse is 5 cm, find the length of the shorter leg.

One leg x, the other leg x-1 , hypotenuse 5

Equation (Pythagoras' theorem) $x^2 + (x-1)^2 = 5^2 \rightarrow x^2 + x^2 - 2x + 1 = 25$

$$2x^2 - 2x - 24 = 0 \rightarrow x^2 - x - 12 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 48}}{2} = \frac{1 \pm 7}{2} = \begin{cases} 4 \\ -2 \leftarrow \text{NO!} \end{cases}$$



Answer: One leg is 4 cm long and the other leg 3 cm

4. The product of two consecutive negative integers is 1260. What are the numbers?

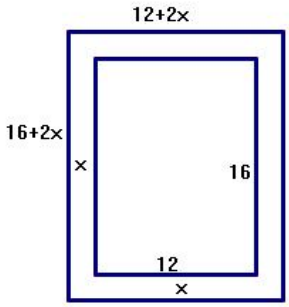
Two consecutive integers x, x + 1

Equation: $x(x+1) = 1260 \rightarrow x^2 + x - 1260 = 0$

$$x = \frac{-1 \pm \sqrt{1 + 4 \cdot 1260}}{2} = \frac{-1 \pm 71}{2} = \begin{cases} 35 \leftarrow \text{NO} \\ -36 \leftarrow \text{YES} \end{cases}$$

Answer: the numbers are -36 and -35

5. A garden measuring 12 metres by 16 metres is to have a pedestrian pathway installed all around it, increasing the total area to 285 square metres. What will be the width of the pathway?



Equation:

$$(2x+12)(2x+16)=285 \rightarrow 4x^2+56x-93=0$$

$$x=\frac{-56\pm\sqrt{3136+4\cdot 4\cdot 93}}{8}$$

$$x=\frac{-56\pm 68}{8}=\begin{cases}\frac{3}{2}\\ -\frac{31}{2}\leftarrow NO\end{cases}$$

Answer: The width of the pathway is 1.5 metres

6. Find the fraction which becomes $\frac{1}{2}$ when the denominator is increased by 5 and is equal to $\frac{1}{2}$ when the numerator is diminished by 4.

X numerator, y denominator $\rightarrow$ fraction $\frac{x}{y}$

$$\text{Equations}\rightarrow \left.\begin{array}{l}\frac{x}{y+5}=\frac{1}{2}\\ \frac{x-4}{y}=\frac{1}{3}\end{array}\right\}\rightarrow \left.\begin{array}{l}2x=y+5\\ 3x-12=y\end{array}\right\}\rightarrow \left.\begin{array}{l}y=2x-5\\ 3x-12=2x-5\end{array}\right\}$$

$$3x-2x=12-5\Rightarrow x=7\Rightarrow y=2\cdot 7-5=14-5=9$$

Answer: the fraction is $\frac{7}{9}$

7. A lady has 50 cents and 2€ coins in her purse. She has 90 coins in all and their total value is 105€. How many 2€ coins does she have?

2€ coins x, 50 cents coins y

$$\text{Equations}\rightarrow \left.\begin{array}{l}x+y=90\\ 2x+0.50y=105\end{array}\right\}$$

$$\rightarrow \left.\begin{array}{l}x=90-y\\ 2x+0.5y=105\end{array}\right\}\rightarrow 2(90-y)+0.5y=105\rightarrow 180-2y+0.5y=105$$

$$180-105=2y-0.5y\rightarrow 75=1.5y\rightarrow y=50\Rightarrow x=90-y=90-50=40$$

Answer: She has 40 2€ coins and 50 coins of 50 cents

8. A mixture containing 6% boric acid is to be mixed with 4 litres of a mixture that is 15% acid in order to obtain a solution that is 12% acid. How much of the 6% solution must be used?

	1st solution	2nd solution	Mixture
litres	4	x – 4	x
percentage	15%	6%	12%

Equation:

$4 \cdot 15 + (x-4)6 = 12x \rightarrow 60 + 6x - 24 = 12x \Rightarrow 6x = 36 \Rightarrow x = 6$

Answer: We have to use x – 4 = 6 – 4 = 2 litres of the 6% solution

9. The owner of a candy store mixes two types of candies. She decides to create a 20-pound mixture of raspberry-flavored gumdrops and cherry-flavored jelly beans. The gumdrops sell for \$0.95 per pound and the jelly beans sell for \$1.20 per pound. She plans to sell the mix for \$1.10 per pound. How many pounds of each candy should she use in her mix?

	gumdrops	jelly beans	Mixture
pounds	x	20 - x	20
Price/pound	\$0.95	\$1.20	\$1.10

Equation:

$0.95x + 1.20(20 - x) = 1.10 \cdot 20 \rightarrow 0.95x + 24 - 1.20x = 22$

$0.95x - 1.20x = 22 - 24 \Rightarrow -0.25x = -2 \Rightarrow x = 8$

Answer: She has to mix 8 pounds of gumdrops with 12 pounds of jelly beans

10. A father is 28 years older than his daughter. In 2 years' time he will be 3 times as old as his daughter. Find their present ages.

Age	Now	In 2 years' time
father	x + 28	x + 28 + 2
daughter	x	x + 2

Equation: $x + 28 + 2 = 3(x + 2) \Rightarrow x + 30 = 3x + 6 \Rightarrow 2x = 24 \Rightarrow x = 12$

Answer: The daughter is 12 years old and her father is 40 years old