EXAMEN INTEGRALES 1

Calcula las siguientes integrales:

1) $\int \frac{2+x^2}{1+x^2}dx$

2) $\int \frac{-2}{\sqrt{16-x^2}}dx$

3) $\int \cos^2 x\, dx$

4) $\int tg^2 5x\, dx$

5) $\int \left(x^3+2\right) e^x dx$

6) $\int x\, arctg 2x\, dx$

7) $\int sen^2 x \cdot \cos x\, dx$

8) $\int \ln^2 x\, dx$

9) $\int (x-1)\sqrt{x^2-2x}\, dx$

10) $\int e^{-x} sen\, 4x\, dx$

PUNTUACIÓN: 1 punto cada integral

SOLUCIONES

1) $\int \frac{2+x^2}{1+x^2}dx = \int \frac{1}{1+x^2}dx + \int \frac{1+x^2}{1+x^2}dx = \text{arctg } x + x + C$

2) $\int \frac{-2}{\sqrt{16-x^2}}dx = \int \frac{\frac{-2}{4}}{\sqrt{\frac{16}{16}-\frac{x^2}{16}}}dx = -2\int \frac{1/4}{\sqrt{1-\left(\frac{x}{4}\right)^2}}dx = -2\text{arcsen}\left(\frac{x}{4}\right) + C$

3) $\int \cos^2 x\, dx = \int \cos x \cdot \cos x\, dx = \text{sen } x \cos x - \int \text{sen } x \cdot (-\text{sen } x)dx = (1)$

$\left.\begin{array}{l} u = \cos x \\ dv = \cos x\, dx \end{array}\right\} \begin{array}{l} du = -\text{sen } x\, dx \\ v = \text{sen} x \end{array}$

(1)= $\text{sen } x \cos x + \int \text{sen}^2 x\, dx = \text{sen } x \cos x + \int (1-\cos^2 x)dx \rightarrow$

$\int \cos^2 x\, dx = \text{sen } x \cos x + x - \int \cos^2 x dx \rightarrow 2\int \cos^2 x\, dx = \text{sen } x \cos x + x$

$\rightarrow \int \cos^2 x\, dx = \frac{\text{sen } x \cos x + x}{2} + C$

4) $\int \text{tg}^2 5x\, dx = \int \frac{\text{sen}^2 5x}{\cos^2 5x}dx = \int \frac{1-\cos^2 5x}{\cos^2 5x}dx = \int \frac{1}{\cos^2 5x}dx - \int dx =$

$= \frac{1}{5}\int \frac{5}{\cos^2 5x}dx - x = \frac{1}{5}\text{tg } 5x - x + C$

5) $\int (x^3+2)\, e^x dx = (x^3+2)e^x - \int 3x^2 e^x dx = (x^3+2)e^x - 3\left[x^2 e^x - 2\int x e^x dx\right] =$

$\left.\begin{array}{l} u = x^3+2 \\ dv = e^x dx \end{array}\right\} \begin{array}{l} du = 3x^2 dx \\ v = e^x \end{array}$ $\quad$ $\left.\begin{array}{l} u = x^2 \\ dv = e^x dx \end{array}\right\} \begin{array}{l} du = 2x dx \\ v = e^x \end{array}$ $\quad$ $\left.\begin{array}{l} u = x \\ dv = e^x dx \end{array}\right\} \begin{array}{l} du = dx \\ v = e^x \end{array}$

$\int (x^3+2)\, e^x dx = (x^3+2)e^x - 3x^2 e^x + 6\left[x e^x - \int e^x dx\right]$

$\int (x^3+2)\, e^x dx = (x^3+2)e^x - 3x^2 e^x + 6x e^x - 6e^x + C = (x^3 - 3x^2 + 6x - 4)e^x + C$

6) $\int x \text{ arctg} 2x\, dx = \frac{x^2}{2}\text{arctg} 2x - \int \frac{x^2}{1+4x^2}dx = \frac{x^2}{2}\text{arctg} 2x - \frac{1}{4}\int \frac{4x^2}{1+4x^2}dx$

$\left.\begin{array}{l} u = \text{arctg} 2x \\ dv = x\, dx \end{array}\right\} \begin{array}{l} du = \frac{2}{1+(2x)^2}dx \\ v = \frac{x^2}{2} \end{array}$

$\int x \text{ arctg} 2x\, dx = \frac{x^2}{2}\text{arctg} 2x - \frac{1}{4}\int \frac{4x^2+1-1}{1+4x^2}dx = \frac{x^2}{2}\text{arctg} 2x - \frac{1}{4}\int dx +$

$+ \frac{1}{4}\int \frac{1}{1+(2x)^2}dx = \frac{x^2}{2}\text{arctg} 2x - \frac{1}{4}x + \frac{1}{4}\cdot\frac{1}{2}\int \frac{2}{1+(2x)^2}dx$

$$\int x\ \text{arctg}\,2x\ dx = \frac{x^2}{2}\text{arctg}\,2x - \frac{1}{4}x + \frac{1}{8}\text{arctg}\,2x + C =$$
$$= \left(\frac{4x^2+1}{8}\right)\text{arctg}\,2x - \frac{x}{4} + C$$

7) $\int \text{sen}^2 x \cdot \cos x\ dx = \int t^2 dt = \frac{t^3}{3} + C = \frac{\text{sen}^3 x}{3} + C$

$t = \text{sen}\ x \to dt = \cos x\ dx$

8) $\int \ln^2 x\ dx = \ln x(x\ln x - x) - \int \frac{x\ln x - x}{x}dx = x\ln^2 x - x\ln x - \int \ln x dx + \int dx$

$$\left.\begin{array}{l} u = \ln x \\ dv = \ln x dx \end{array}\right\} \begin{array}{l} du = \frac{1}{x}dx \\ v = \int \ln x dx = (*) \end{array} \left\{\begin{array}{l} (*) = x\ln x - \int \frac{x}{x}dx = x\ln x - x \\ \left.\begin{array}{l} u = \ln x \\ dv = dx \end{array}\right\} \begin{array}{l} du = \frac{1}{x}dx \\ v = x \end{array} \end{array}\right.$$

$$\int \ln^2 x\ dx = x\ln^2 x - x\ln x - x\ln x + x + x + C = x\ln^2 x - 2x\ln x + 2x + C$$

9) $\int (x-1)\sqrt{x^2-2x}\ dx = \frac{1}{2}\int 2(x-1)\sqrt{x^2-2x}dx = \frac{1}{2}\int \sqrt{t}dt = \frac{1}{2}\int t^{\frac{1}{2}}dt$

$t = x^2 - 2x \to dt = (2x-2)dx = 2(x-1)dx$

$$\int (x-1)\sqrt{x^2-2x}\ dx = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}t^{\frac{3}{2}} + C = \frac{1}{3}\sqrt{(x^2-2x)^3} + C$$

10) $\int e^{-x}\text{sen}\ 4x\ dx = -\frac{1}{4}e^{-x}\cos 4x - \frac{1}{4}\int e^{-x}\cos 4x dx =$

$$\left.\begin{array}{l} u = e^{-x} \\ dv = \text{sen}\,4x\ dx \end{array}\right\} \begin{array}{l} du = -e^{x}dx \\ v = -\frac{1}{4}\cos 4x \end{array} \qquad \left.\begin{array}{l} u = e^{-x} \\ dv = \cos 4x dx \end{array}\right\} \begin{array}{l} du = -e^{-x}dx \\ v = \frac{1}{4}\text{sen}\,4x \end{array}$$

$$= -\frac{1}{4}e^{-x}\cos 4x - \frac{1}{4}\left[\frac{1}{4}e^{-x}\text{sen}\,4x + \frac{1}{4}\int e^{-x}\text{sen}\,4x\ dx\right]$$

Llamamos $I = \int e^{-x}\text{sen}\ 4x\ dx$ y tenemos que:

$$I = -\frac{1}{4}e^{-x}\cos 4x - \frac{1}{16}e^{-x}\text{sen}\,4x - \frac{1}{16}I \to I + \frac{I}{16} = -\frac{1}{4}e^{-x}\cos 4x - \frac{1}{16}e^{-x}\text{sen}\,4x$$

$$\frac{17I}{16} = -\frac{1}{4}e^{-x}\cos 4x - \frac{1}{16}e^{-x}\text{sen}\,4x \to I = \frac{16}{17}\left[-\frac{1}{4}e^{-x}\cos 4x - \frac{1}{16}e^{-x}\text{sen}\,4x\right] + C$$