IES MURILLO

Maths 4th ESO

EXAM 1_2 (Surds – Algebra)

1) Work out and simplify: (2 p)

a) $\frac{\sqrt{3}+2}{\sqrt{3}-1} - \frac{\sqrt{3}}{\sqrt{3}+1} =$

b) $10\sqrt{7} - 8\sqrt{28} - 3\sqrt{8} + 2\sqrt{63} =$

2) Work out and simplify: (2.5 p)

a) $\frac{(x-1)^2}{2} \cdot \frac{1}{x^2-1} - \frac{3x}{(x+1)^2} =$

b) $\left(\frac{3}{2x} - \frac{2x}{x+2}\right) \div \frac{x-1}{x^2+2x} =$

3) Solve the equations: (2.5 p)

a) $\frac{4-x}{x^2+2x+1} - \frac{2-x}{x+1} = 2$

b) $x^3 - 8x^2 + 20x - 16 = 0$

4) Find two consecutive numbers whose squares added together equal 145. (1.5 p)

5) A rectangle is 4 times as long as it is wide. If the length is increased by 4 cm and the width is decreased by 1 cm, the area will be 60 cm^2. What were the dimensions of the original rectangle? (1.5 p)

SOLUTION

1) Work out and simplify:

$$a)\ \frac{\sqrt{3}+2}{\sqrt{3}-1}-\frac{\sqrt{3}}{\sqrt{3}+1}=\frac{(\sqrt{3}+2)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}-\frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{(\sqrt{3}+2)(\sqrt{3}+1)}{\sqrt{3}^2-1^2}-$$

$$-\frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{3}^2-1^2}=\frac{3+2\sqrt{3}+\sqrt{3}+2}{2}-\frac{3-\sqrt{3}}{2}=\frac{4\sqrt{3}+2}{2}=2\sqrt{3}+1$$

$$b)10\sqrt{7}-8\sqrt{28}-3\sqrt{8}+2\sqrt{63}=10\sqrt{7}-8\sqrt{2^2\cdot 7}-3\sqrt{2^3}+2\sqrt{3^2\cdot 7}=$$

$$=10\sqrt{7}-16\sqrt{7}-6\sqrt{2}+6\sqrt{7}=-6\sqrt{2}$$

2) Work out and simplify:

$$a)\ \frac{(x-1)^2}{2}\cdot\frac{1}{x^2-1}-\frac{3x}{(x+1)^2}=\frac{(x-1)(x-1)}{2(x-1)(x+1)}-\frac{3x}{(x+1)^2}=\frac{(x-1)}{2(x+1)}-\frac{3x}{(x+1)^2}=$$

$$=\frac{(x-1)(x+1)}{2(x+1)^2}-\frac{6x}{2(x+1)^2}=\frac{x^2-6x-1}{2(x+1)^2}$$

$$b)\ \left(\frac{3}{2x}-\frac{2x}{x+2}\right)\div\frac{x-1}{x^2+2x}=\left(\frac{3(x+2)}{2x(x+2)}-\frac{2x\cdot 2x}{2x(x+2)}\right)\div\frac{x-1}{x^2+2x}=$$

$$=\left(\frac{3x+6-4x^2}{2x(x+2)}\right)\div\frac{x-1}{x^2+2x}=\frac{-4x^2+3x+6}{2x(x+2)}\cdot\frac{x(x+2)}{x-1}=\frac{-4x^2+3x+6}{2(x-1)}$$

3) Solve the equations:

$$a)\ \frac{4-x}{x^2+2x+1}-\frac{2-x}{x+1}=2\ \rightarrow LCD=(x+1)^2$$

$$\frac{4-x}{(x+1)^2}-\frac{(2-x)(x+1)}{(x+1)^2}=\frac{2(x+1)^2}{(x+1)^2}\rightarrow 4-x-(2x-x^2+2-x)=2(x^2+2x+1)$$

$$4-x-2x+x^2-2+x=2x^2+4x+2\rightarrow x^2+6x=0\rightarrow x(x+6)=0\Rightarrow\begin{cases}x=0\\x=-6\end{cases}$$

$$b)\ x^3-8x^2+20x-16=0;\ \text{Factors of 16: } \pm1,\pm2,\pm4,\pm8,\pm16$$

$$P(1)=1^3-8+20-16\neq 0;\ P(-1)=-1-8-20-16\neq 0;$$

$$P(2)=2^3-8\cdot 4+20\cdot 2-16=0$$

	1	-8	+20	-16
2		2	-12	+16
	1	-6	+8	0

$$x^2-6x+8=0\rightarrow x=\frac{6\pm\sqrt{4}}{2}=\begin{cases}4\\2\end{cases}$$

$$x^3 - 8x^2 + 20x - 16 = 0 \rightarrow (x-2)^2(x-4) = 0 \Rightarrow x = \begin{cases} 2 \\ 4 \end{cases} \text{ ROOTS}$$

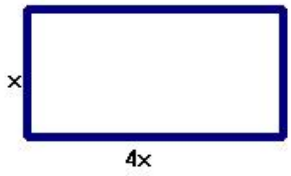
4) Find two consecutive numbers whose squares added together equal 145.
Numbers: x, x+1

$$x^2 + (x+1)^2 = 145 \Rightarrow x^2 + x^2 + 2x + 1 = 145 \Rightarrow 2x^2 + 2x - 144 = 0$$

$$x^2 + x - 72 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+288}}{2} = \frac{-1 \pm 17}{2} = \begin{cases} 8 \\ -9 \end{cases}$$

Answer: Consecutive numbers $\begin{cases} 8, 9 \\ -9, -8 \end{cases}$

5) A rectangle is 4 times as long as it is wide. If the length is increased by 4 cm and the width is decreased by 1 cm, the area will be 60 cm^2. What were the dimensions of the original rectangle?



Length new rectangle 4x + 4
Width new rectangle x – 1
Area new rectangle 60

$$(4x+4)(x-1) = 60 \Rightarrow 4x^2 + 4x - 4x - 4 = 60 \Rightarrow 4x^2 - 64 = 0 \Rightarrow x = \pm\sqrt{16} = \pm 4$$

So x = 8 cm, Dimensions of the original rectangle: 4 cm x 16 cm