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Maths 4th ESO

EXAM 1_3 (Algebraic Fractions - Equations)

1. Work out and simplify: (2.5 points)

a) $\left(1+\frac{1}{x-1}\right):\frac{x^2+x}{x^2-1}=$

b) $\frac{x^2-3x}{x+2}:\frac{x^2-5x+6}{x^2-4}=$

2. Solve the following: (1.5 points each equation)

a) $\frac{x}{x-2}-\frac{2}{x-2}=\frac{6}{x}$

b) $x+\sqrt{3x+10}=6$

c) $x^4+x^3-4x^2-4x=0$

d) $\left(x^2+3x\right)^2-3\left(2x^3+4x^2-2\right)=2-4(x-2)(x+2)$

e) $\sqrt{2x}-\sqrt{x+1}=-1$

SOLUTION

1. a) $\left(1+\frac{1}{x-1}\right):\frac{x^2+x}{x^2-1}=\left(\frac{x-1+1}{x-1}\right):\frac{x^2+x}{x^2-1}=\frac{x(x^2-1)}{(x-1)(x^2+x)}=$

$=\frac{x(x-1)(x+1)}{(x-1)\cdot x(x+1)}=1$

b) $\frac{x^2-3x}{x+2}:\frac{x^2-5x+6}{x^2-4}=\frac{(x^2-3x)(x^2-4)}{(x+2)(x^2-5x+6)}$

$=\frac{x(x-3)(x-2)(x+2)}{(x+2)(x-3)(x-2)}=x$

$x^2-3x=x(x-3)$ $\qquad$ $x^2-5x+6=0\Rightarrow x=\frac{5\pm\sqrt{25-24}}{2}=\begin{cases}3\\2\end{cases}$

$x^2-4=(x+2)(x-2)$ $\qquad$ $x^2-5x+6=(x-3)(x-2)$

2. a) $\frac{x}{x-2}-\frac{2}{x-2}=\frac{6}{x}\Rightarrow\frac{x^2-2x}{x(x-2)}=\frac{6(x-2)}{x(x-2)}\Rightarrow x^2-8x+12=0\Rightarrow x=\begin{cases}6\\2\end{cases}$

$x=2$ is not a solution, this value would cause division by zero in the original equation. Since the solution is **x = 6**

b) $x+\sqrt{3x+10}=6\Rightarrow\sqrt{3x+10}=6-x\Rightarrow 3x+10=(6-x)^2$

$3x+10=36+x^2-12x\Rightarrow 0=x^2-15x+26$

$x=\frac{15\pm\sqrt{225-104}}{2}=\frac{15\pm\sqrt{121}}{2}=\frac{15\pm11}{2}\quad\rightarrow\quad\begin{cases}x=13\\x=2\end{cases}$

Check:

$x=13\quad\rightarrow\quad 13+\sqrt{49}=13+7=20\neq 6\quad\rightarrow\quad x=13$ no!

$x=2\quad\rightarrow\quad 2+\sqrt{16}=2+4=6\quad\rightarrow\quad x=2$ yes!

SOLUTION: **x = 2**

c) $x^4+x^3-4x^2-4x=0$, we factorize the polynomial:

$x^4+x^3-4x^2-4x=x(x^3+x^2-4x-4)$

$Div(4) = \pm 1, \pm 2, \pm 4$

$P(1) = 1 + 1 - 4 - 4 \neq 0$ $\qquad$ $P(-1) = -1 + 1 + 4 - 4 = 0$

	1	+1	- 4	- 4
-1		-1	0	+4
	1	0	- 4	0

$x^2 - 4 = 0$

$x = \sqrt{4} = \pm 2$

EQUATION: $x(x+1)(x-2)(x+2) = 0$

SOLUTION: $x = 0;\ x = -1;\ x = 2;\ x = -2$

d) $(x^2 + 3x)^2 - 3(2x^3 + 4x^2 - 2) = 2 - 4(x-2)(x+2)$

$x^4 + 6x^3 + 9x^2 - 6x^3 - 12x^2 + 6 = 2 - 4(x^2 - 4)$

$x^4 + 9x^2 - 12x^2 + 6 = 2 - 4x^2 + 16 \rightarrow x^4 + x^2 - 12 = 0$ biquadratic

$$z = x^2 \rightarrow z^2 + z - 12 = 0 \rightarrow z = \frac{-1 \pm \sqrt{1+48}}{2} = \begin{cases} 3 \\ -4 \end{cases} \rightarrow x = \begin{cases} \pm\sqrt{3} \\ \pm\sqrt{-4} \end{cases}$$

SOLUTION: $x = \pm\sqrt{3}$

e) $\sqrt{2x} - \sqrt{x+1} = -1 \Rightarrow \sqrt{2x} + 1 = \sqrt{x+1} \Rightarrow \left(\sqrt{2x} + 1\right)^2 = \left(\sqrt{x+1}\right)^2$

$\left(\sqrt{2x}\right)^2 + 2\sqrt{2x} + 1 = x + 1 \Rightarrow 2\sqrt{2x} = x + 1 - 2x - 1 \Rightarrow 2\sqrt{2x} = -x$

$$\left(2\sqrt{2x}\right)^2 = (-x)^2 \Rightarrow 4 \cdot 2x = x^2 \Rightarrow 8x - x^2 = 0 \Rightarrow x(8-x) = 0 \Rightarrow \begin{cases} x = 0 \\ x = 8 \end{cases}$$

Check:

$\sqrt{0} - \sqrt{0+1} = 0 - 1 = -1$ YES $\qquad$ $\sqrt{16} - \sqrt{8+1} = 4 - 3 = 1 \neq -1$ NO

SOLUTION **x = 0**