IES MURILLO

Maths 4th ESO

EXAM 2_2 (Functions 1)

Name:.....

(2 points each)

1. A toy rocket is launched from a platform that is 16 feet above the ground. The height, h, in feet, of the rocket x seconds after launch is given by the formula $h(x) = -16x^2 + 126x + 16$. Draw the graph and work out:

a) The height of the rocket 2 seconds after launching.

b) The maximum height the rocket reaches.

c) After how many seconds is the rocket 114 feet above the ground?

d) When will the rocket hit the ground?

2. Solve by substitution and graphically the simultaneous equations:

$$\left.\begin{array}{r} x^2 + 16 = y + 10x \\ x - y = 2 \end{array}\right\}$$ (Remember: Do not use data table!)

3. Find the domain of the following functions:

$f(x) = x^3 - 2x + 7$ $\quad$ $g(x) = \sqrt{2x+5}$ $\quad$ $h(x) = \dfrac{x+1}{x^2 - 10x + 16}$

4. A function is given as: $f(x) = \begin{cases} x^2 & x < 0 \\ 3 & 0 < x < 2 \\ 2x - 1 & x \geq 2 \end{cases}$

a) Find f(-2), f(0), f(1), f(2), f(3)

b) Sketch the function

d) What are its domain and range?

e) Intervals of increase

f) Continuity

5. Given the equation of the parabola: $f(x) = -x^2 + 2x - 5$

a) Find its vertex, axis of symmetry, its intersections with the x axis and the y axis, and draw its graph.

b) Domain and Range

SOLUTION

1. A toy rocket is launched from a platform that is 16 feet above the ground. The height, h, in feet, of the rocket x seconds after launch is given by the formula $h(x) = -16x^2 + 126x + 16$. Draw the graph and work out:

a) The height of the rocket 2 seconds after launching.

b) The maximum height the rocket reaches.

c) After how many seconds is the rocket 114 feet above the ground?

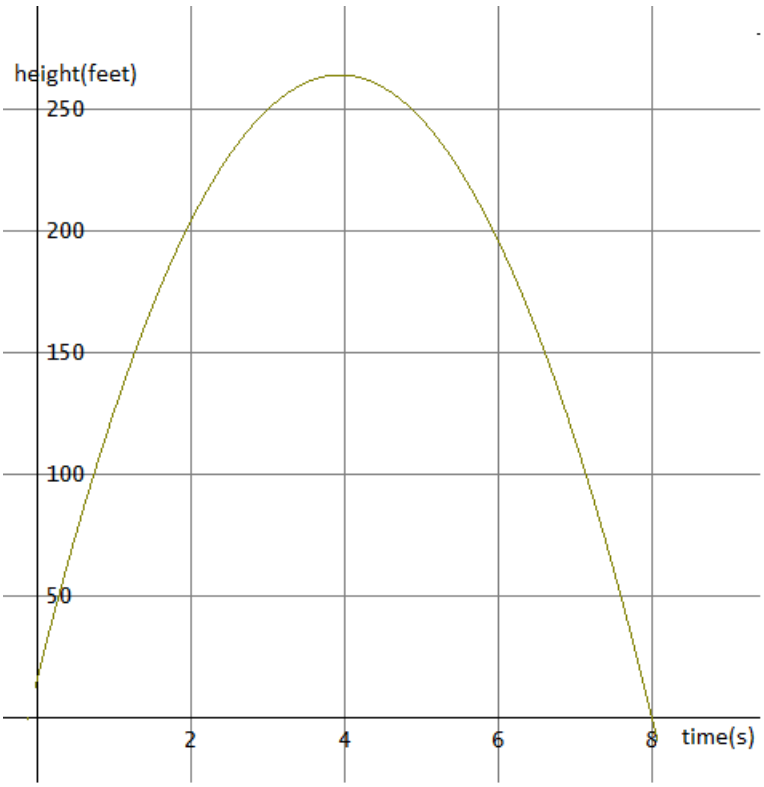
d) When will the rocket hit the ground?

It is a parabola, open downward

Vertex $x = -\frac{126}{-32} = 3.9375 \rightarrow y = 264.0625 \rightarrow V(3.9375, 264.0625)$

x and y intercepts: $x = 0 \rightarrow y = 16$

$$y = 0 \rightarrow -16x^2 + 126x + 16 = 0 \rightarrow x = \frac{-126 \pm \sqrt{16900}}{-32} = \frac{-126 \pm 130}{-32} = \begin{cases} -\frac{1}{8} \text{ no (time)} \\ 8 \end{cases}$$



a)

$h(2) = -16 \cdot 2^2 + 126 \cdot 2 + 16$

$h(2) = 204$ feet, 2 seconds after launch

b) Maximum height (in the vertex)

264.0625 feet

c) $y = 114 \rightarrow -16x^2 + 126x + 16 = 114 \; -16x^2 + 126x - 98 = 0$

$$x = \frac{-126 \pm \sqrt{126^2 - 6272}}{-32} = \frac{-126 \pm \sqrt{9604}}{-32} = \frac{-126 \pm 98}{-32} \begin{cases} 0.875 \\ 7 \end{cases}$$

After 0.875 seconds and after 7 seconds the rocket is 114 feet above the ground

d) The rocket hit the ground in the x-intercepts, 8 second after launch.

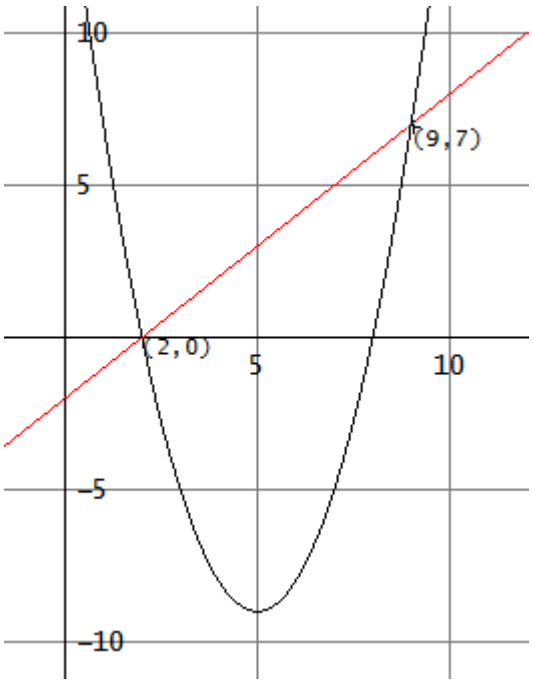
2. Solve by substitution and graphically the simultaneous equations:

$$\left.\begin{array}{r} x^2+16=y+10x \\ x-y=2 \end{array}\right\} \rightarrow y=x-2 \rightarrow x^2+16=x-2+10x \rightarrow x^2-11x+18=0$$

$$x=\frac{11\pm\sqrt{121-72}}{2}=\frac{11\pm 7}{2}=\begin{cases} 9\rightarrow y=9-2=7 \\ 2\rightarrow y=2-2=0 \end{cases} \rightarrow SOL\begin{cases} x=9,\ y=7 \\ x=2,\ y=0 \end{cases}$$

Graphically:

$$\left.\begin{array}{r} y=x^2-10x+16 \\ y=x-2 \end{array}\right\} \begin{array}{l} \rightarrow \text{Vertex } x=\frac{10}{2}=5\rightarrow y=-9 \\ \rightarrow \text{slope}=1, y-\text{intercepts - 2} \end{array} \left.\begin{array}{l} \rightarrow \text{Upwards } \cup \\ \rightarrow (0,-2) \end{array}\right\}$$



x- intercepts (parabola)

$$x^2-10x+16=0$$

$$x=\frac{10\pm\sqrt{100-64}}{2}=\frac{10\pm 6}{2}=\begin{cases} 8\rightarrow (8,0) \\ 2\rightarrow (2,0) \end{cases}$$

y- intercepts (parabola)

$$x=0\rightarrow y=16\rightarrow (0,16)$$

3. Find the domain of the following functions:

$$f(x)=x^3-2x+7\rightarrow D(f)=\mathbb{R}$$

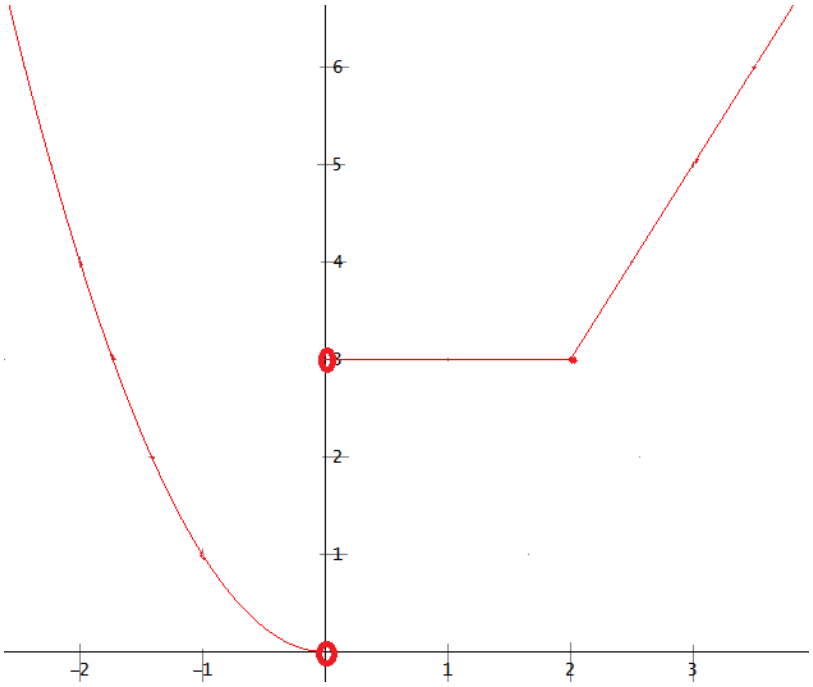
$$g(x)=\sqrt{2x+5}\rightarrow 2x+5\geq 0\rightarrow x\geq -\frac{5}{2}\Rightarrow D(g)=\left[-\frac{5}{2},+\infty\right)$$

$$h(x)=\frac{x+1}{x^2-10x+16}\rightarrow x^2-10x+16=0\rightarrow \begin{cases} x=8 \\ x=2 \end{cases}\Rightarrow D(h)=\mathbb{R}-\{2,8\}$$

4. A function is given as: $f(x)=\begin{cases} x^2 & x<0 \\ 3 & 0<x<2 \\ 2x-1 & x\geq 2 \end{cases}$

a) f(-2)= 4, f(0) It does not exist, f(1)= 3, f(2)= 3, f(3)= 5

b) Sketch the function



c) What are its domain and range?

$D = (-\infty,0) \cup (0,+\infty)$

$R = (0,+\infty)$

d) Intervals of increase:

Increasing $(2,+\infty)$,

Decreasing $(-\infty,0)$,

Constant $(0,2)$

e) Continuity: f(x) is continuous in $(-\infty,0) \cup (0,+\infty)$

The point $x = 0$ is a jump discontinuity

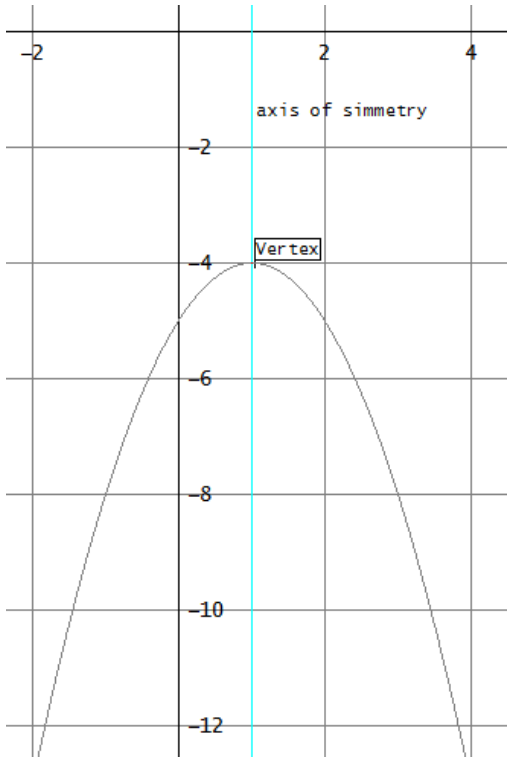
5. $f(x) = -x^2 + 2x - 5$ It is a parabola, open downward

a) Vertex $x = -\frac{2}{-2} = 1 \rightarrow y = -1^2 + 2 - 5 = -4 \rightarrow V(1,-4)$

Axis of symmetry x = 1, intersections with the x axis and the y axis:

x- intercepts $-x^2 + 2x - 5 = 0$

$$x = \frac{-2 \pm \sqrt{4-20}}{-2} = \frac{2 \pm \sqrt{-16}}{-2} = \text{NO}$$



y- intercepts

$x = 0 \rightarrow y = -5 \rightarrow (0,-5)$

Another point (we need it)

if $x = 3 \rightarrow y = -8 \rightarrow (3,-8)$

b) Domain and Range

$D = (-\infty,+\infty)$

$R = (-\infty,-4]$