IES MURILLO

Maths 4th ESO

EXAM 2_3 (Functions 2)

Name:.....

1. Solve by substitution and graphically the simultaneous equations:

$$\left.\begin{array}{r} y-1=\dfrac{2}{x-2} \\ 2x = y+3 \end{array}\right\}$$ (2 points)

2. Sketch the graphs of the following functions and write their characteristics:

a) $y = 3^{x-1}$ b) $y = \left(\frac{1}{2}\right)^x - 3$ c) $y = log_2(x+2)$ (2.25 p)

3. A person has \$200 to buy stickers. If they cost \$0.5 each, how many stickers can he buy?
   a) Analyse and describe the type of relation between cost per sticker and the number of stickers.
   b) Write the formula and graph the function. (1.5 points)

4. Sketch the graph of the compound function:

$$f(x) = \begin{cases} -2 & x \le -2 \\ 2x+2 & -2 < x < 3 \\ \sqrt{x-3} & x \ge 3 \end{cases}$$ (1.5 points)

   a) Domain and range
   b) Continuity

5. All logarithmic functions ($y = log_a x$) pass through the same point, which point? Why?
   When these functions are decreasing? Why?
   What are the domain and range of these functions ? (1.25 points)

6. In the following equations, find x: (1.5 points)

a) $log_3 \frac{1}{81} = x$ b) $log_2(x^2 - 8) = 0$

c) $log_8 x = \frac{1}{3}$ d) $log_x 49 = -2$

SOLUTION

1. Solve by substitution and graphically the simultaneous equations:

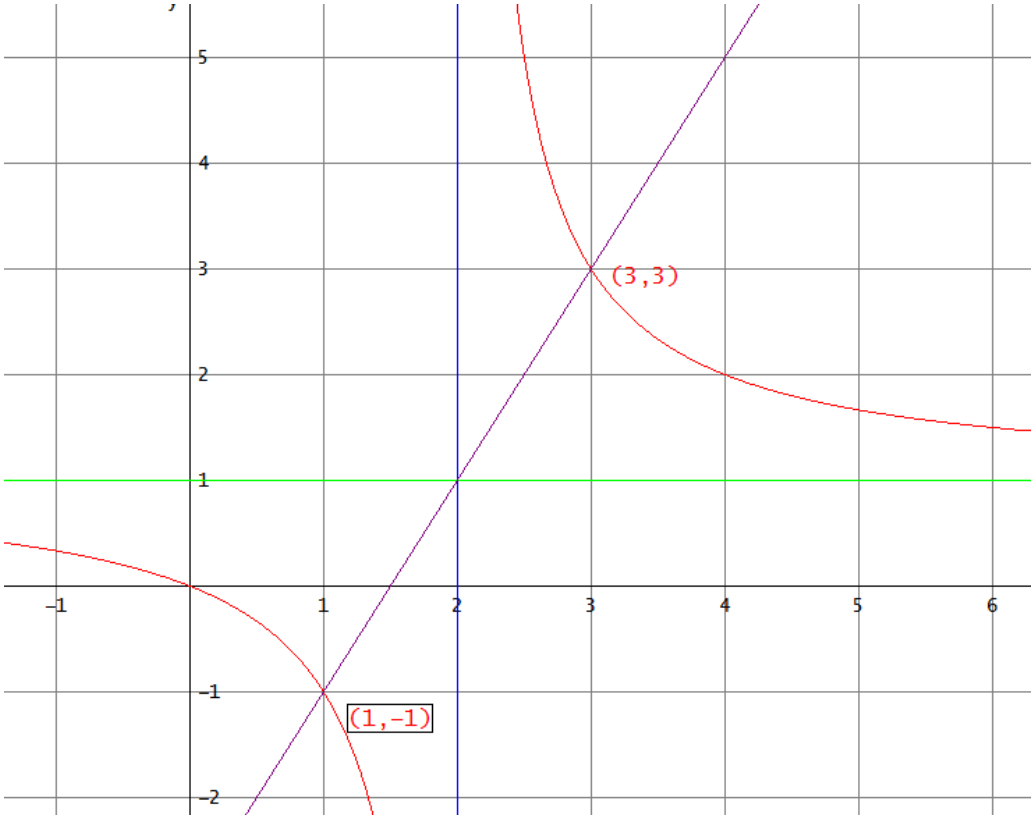
$$\left.\begin{array}{r} y-1=\dfrac{2}{x-2} \\ 2x = y+3 \end{array}\right\} \rightarrow y = 2x-3 \rightarrow 2x-3-1=\frac{2}{x-2} \rightarrow (2x-4)(x-2)=2$$

$$2(x-2)(x-2)=2 \rightarrow (x-2)^2=1 \rightarrow x^2-4x+3=0 \rightarrow x=\begin{cases} 1 \rightarrow y=-1 \\ 3 \rightarrow y=3 \end{cases}$$

Graphically: $\left.\begin{array}{r} y-1=\dfrac{2}{x-2} \\ 2x = y+3 \end{array}\right\} \rightarrow \left.\begin{array}{l} y=\dfrac{2}{x-2}+1 \text{ hyperbola} \\ y=2x-3 \text{ straight line} \end{array}\right\}$

$y=\dfrac{2}{x-2}+1$ hyperbola, asymptotes: horizontal y = 1, vertical x = 2

$y=2x-3 \rightarrow$ straight- line , slope 2, intercepts-y -3



SOLUTION: (3, 3) and (1, -1)

2. Sketch the graphs of the following functions and write its characteristics:

a) $y = 3^{x-1}$

exponential function

base >1,

Dom = $(-\infty,+\infty)$

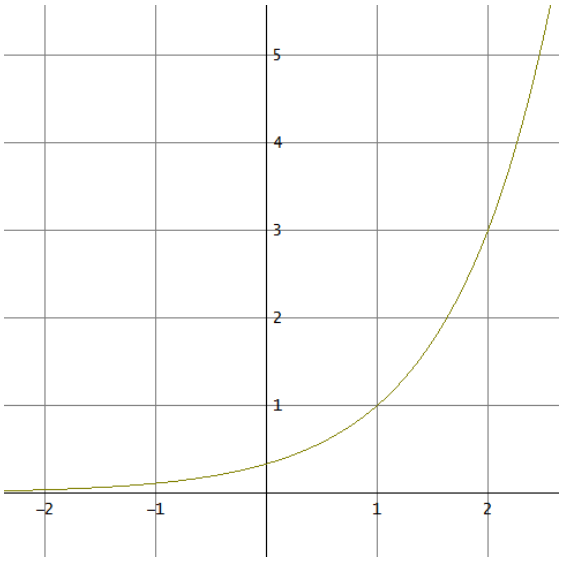
Range = $(0,+\infty)$

Increasing $(-\infty,+\infty)$

point (1,1)

horizontal asymptote x- axis

Continuous



b) $y = \left(\frac{1}{2}\right)^x - 3$

exponential function

base <1,

Dom = $(-\infty,+\infty)$

Range = $(-3,+\infty)$

Decreasing $(-\infty,+\infty)$

point (0,-2)

horizontal asymptote x= -3

c) $y = log_2(x+2)$

logarithmic function

base >1

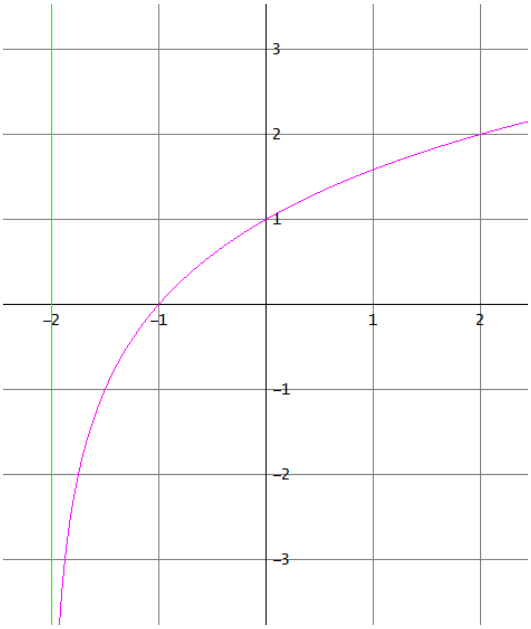
Dom = $(-2,+\infty)$

Range = $(-\infty,+\infty)$

Increasing $(-2,+\infty)$

Point (0,1)

Vertical asymptote y=-2



3. A person has $200 to buy stickers, if they cost $0.5 each. How many stickers can he buy?

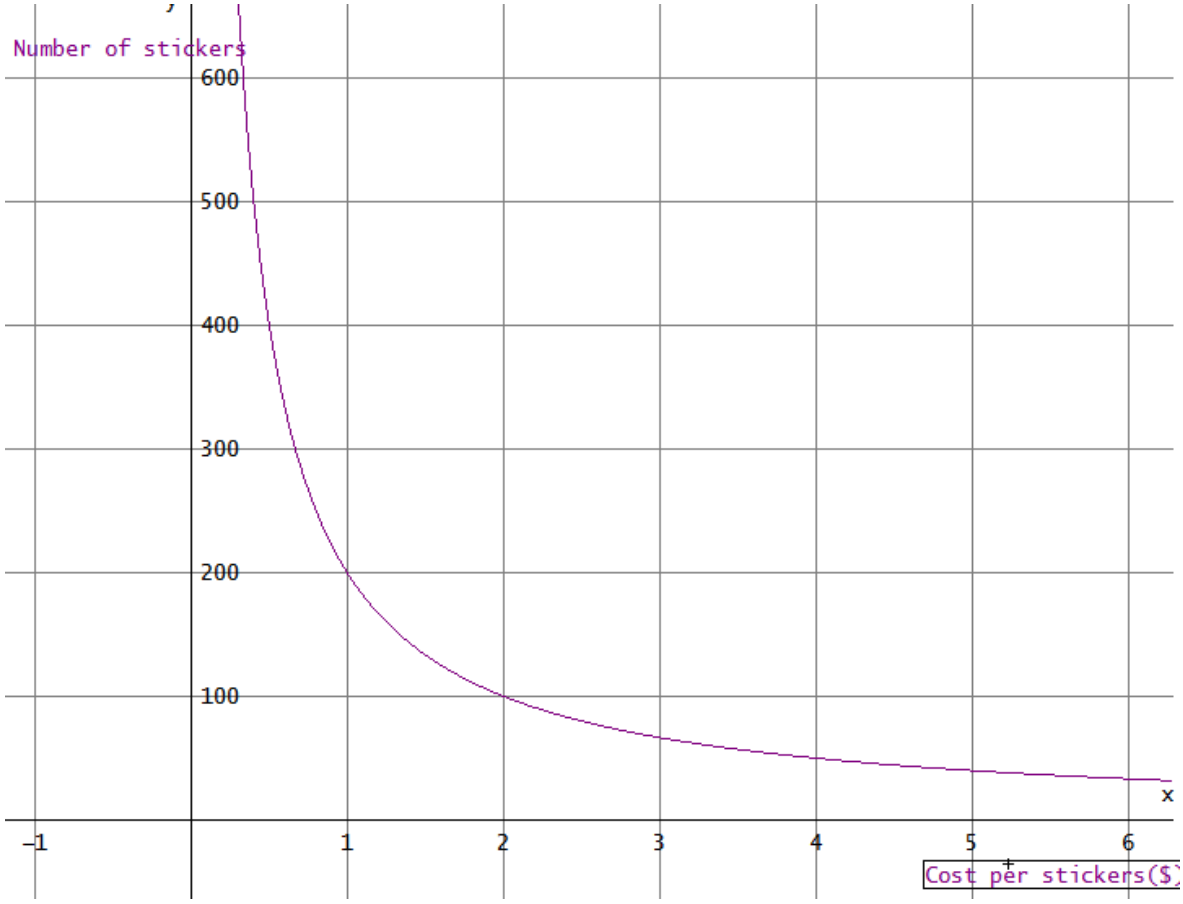
   a) Analyse and describe the type of relation between cost per sticker and the number of stickers.

   He can buy 400 stickers

Cost per sticker ($)	0.5	1	2	4
Number of stickers	400	200	100	50

   It is a inverse proportional function, more price less stickers

   b) Write the formula and graph the function. $y = \frac{200}{x}$



4. Sketch the graph of the compound function:

$$f(x) = \begin{cases} -2 & x \le -2 \\ 2x+2 & -2 < x < 3 \\ \sqrt{x-3} & x \ge 3 \end{cases}$$

   a) Domain and range: Dom = $(-\infty,+\infty)$; Range = $[-2,+\infty)$

   b) Continuity: It is discontinuous in x = 3, it is a jump discontinuity

5. All logarithmic functions ($y = log_a x$) pass through the same point, which point? Why? All these functions pass through the point (1, 0) because $y = log_a 1 = 0 \Leftrightarrow a^0 = 1$

When these functions are decreasing? Why? It depends of the base a, if $a > 1$ the function is increasing, but when the base $0 < a < 1$ the function is decreasing on its domain.

What are the domain and range of these functions? Domain $= [0, +\infty)$ and Range $= (-\infty, +\infty)$

6. In the following equations, find x:

a) $log_3 \frac{1}{81} = x \Leftrightarrow 3^x = \frac{1}{81} \rightarrow 3^x = 3^{-4} \rightarrow x = -4$

b) $log_2 (x^2 - 8) = 0 \Leftrightarrow 2^0 = x^2 - 8 \rightarrow 1 = x^2 - 8 \rightarrow x^2 = 9 \rightarrow x = \pm 3$

c) $log_8 x = \frac{1}{3} \Leftrightarrow 8^{\frac{1}{3}} = x \rightarrow \sqrt[3]{8} = x \rightarrow x = 2$

d) $log_x 49 = -2 \Leftrightarrow x^{-2} = 49 \rightarrow x^{-2} = 7^2 \rightarrow x^{-2} = \left(\frac{1}{7}\right)^{-2} \rightarrow x = \frac{1}{7}$