EXAM 3_2 (Geometry-Trigonometry- Functions)

1. Two men on opposite sides of a TV tower of height 50 m notice the angle of elevation of the top of this tower to be 45° and 65° respectively. Find the distance between the two men. (1.25 p)

2. Find the Domain of the following functions: (1 p)

$$f(x)=\sqrt[3]{\frac{x^3-5x}{x+2}}\;;\qquad g(x)=\sqrt{-x^2-5x+6}$$

3. Suppose that $\sin\alpha=-\frac{2}{5}$ and α lies in quadrant IV. (Don't use a calculator).

a) Draw the angle α

b) Find the other trigonometric ratios for α. (1.25 p)

4. Consider: Triangle ABC with vertices A (-1, 3) B (4, 1) and C (6, 6)

a) Sketch triangle ABC on the Cartesian plane.

b) Show that ABC is an isosceles triangle.

c) Determine the co-ordinates of M, the midpoint of AC.

d) Determine the gradient of AB.

f) Find the equation of the height for AB.

g) Find the equation of the perpendicular bisector of AC. (4 p)

5. Plot the function (don't use a table data): $f(x)=\begin{cases}2 & x<-3\\ -2x-4 & -3<x<2\\ \frac{3}{x-1} & x\geq 2\end{cases}$

And find:

a) Domain and range.

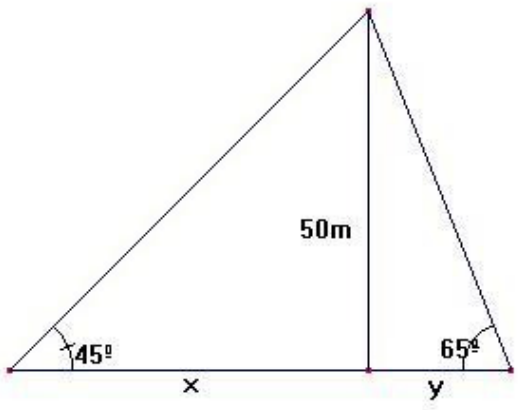
b) Increasing and decreasing intervals.

c) Continuity. (1.5 p)

6. Find the equation of a circle that has a diameter with the endpoints given by the points A(-1 , 2) and B(3 , 2). (1 p)

SOLUTION

1. Two men on opposite sides of a TV tower of height 50 m notice the angle of elevation of the top of this tower to be 45° and 65° respectively. Find the distance between the two men.



$$\tan 45 = \frac{50}{x}; \quad \tan 65 = \frac{50}{y}$$

$$x = \frac{50}{\tan 45} = 50$$

$$y = \frac{50}{\tan 65} = 23.32$$

The distance between the two men is 73.32 metres

2. Find the Domain of the following functions:

$$f(x) = \sqrt[3]{\frac{x^3 - 5x}{x+2}} \qquad Dom(f) = \Re - \{-2\}$$

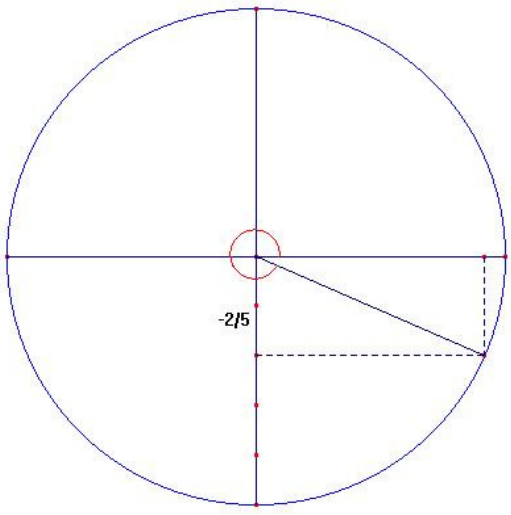
$$g(x) = \sqrt{-x^2 - 5x + 6} \quad \rightarrow -x^2 - 5x + 6 \geq 0 \rightarrow -x^2 - 5x + 6 = 0 \Rightarrow x = \begin{cases} -6 \\ 1 \end{cases}$$

$$Dom(g) = [-6, 1]$$

3. Suppose that $\sin\alpha = -\frac{2}{5}$ and α lies in quadrant IV. (Don't use a calculator).

a) Draw the angle α

b) Find the other trigonometric ratios for α.



$$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = 1 - \frac{4}{25} = \frac{21}{25}$$

$$\cos\alpha = +\frac{\sqrt{21}}{5} \rightarrow \sec\alpha = \frac{5}{\sqrt{21}}$$

$$\tan\alpha = -\frac{2}{\sqrt{21}} \rightarrow \cot\alpha = -\frac{\sqrt{21}}{2}$$

$$\operatorname{cosec}\alpha = -\frac{5}{2}$$

4. Consider: Triangle ABC with vertices A (-1, 3) B (4, 1) and C (6, 6)

a) Sketch triangle ABC on the Cartesian plane.

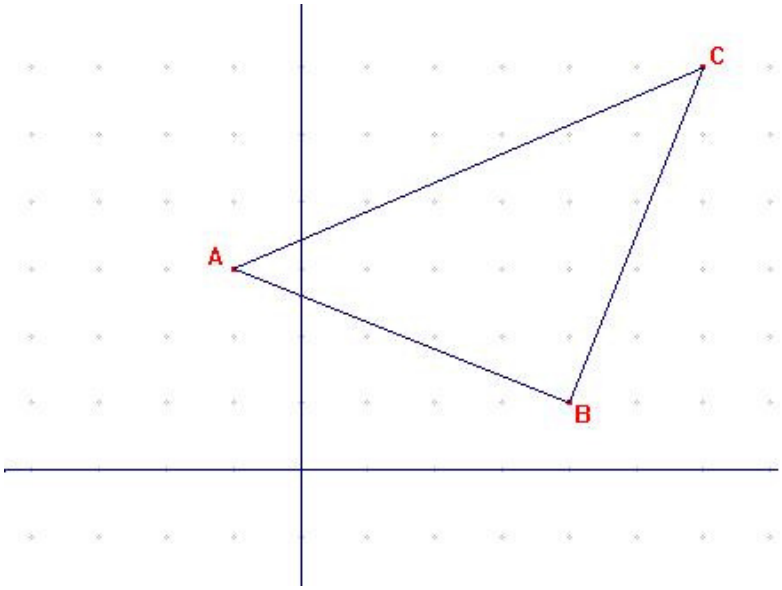
b) Show that ABC is an isosceles triangle.

c) Determine the co-ordinates of M, the midpoint of AC.

d) Determine the gradient of AB.

e) Find the equation of the height for AB.

f) Find the equation of the perpendicular bisector of AC.



b) $d(A,B)=\sqrt{(4+1)^2+(1-3)^2}=\sqrt{29}$ u

$d(A,C)=\sqrt{(6+1)^2+(6-3)^2}=\sqrt{58}$ u congruent sides AB and BC

$d(C,B)=\sqrt{(6-4)^2+(6-1)^2}=\sqrt{29}$ u

c) $M\rightarrow\left(\frac{-1+6}{2},\frac{3+6}{2}\right)\rightarrow M\left(\frac{5}{2},\frac{9}{2}\right)$

d) $m_{AB}=\frac{y_2-y_1}{x_2-x_1}=\frac{1-3}{4+1}=-\frac{2}{5}$

e)Height for AB: point C(6,6) and perpendicular to $AB\rightarrow m=\frac{5}{2}$

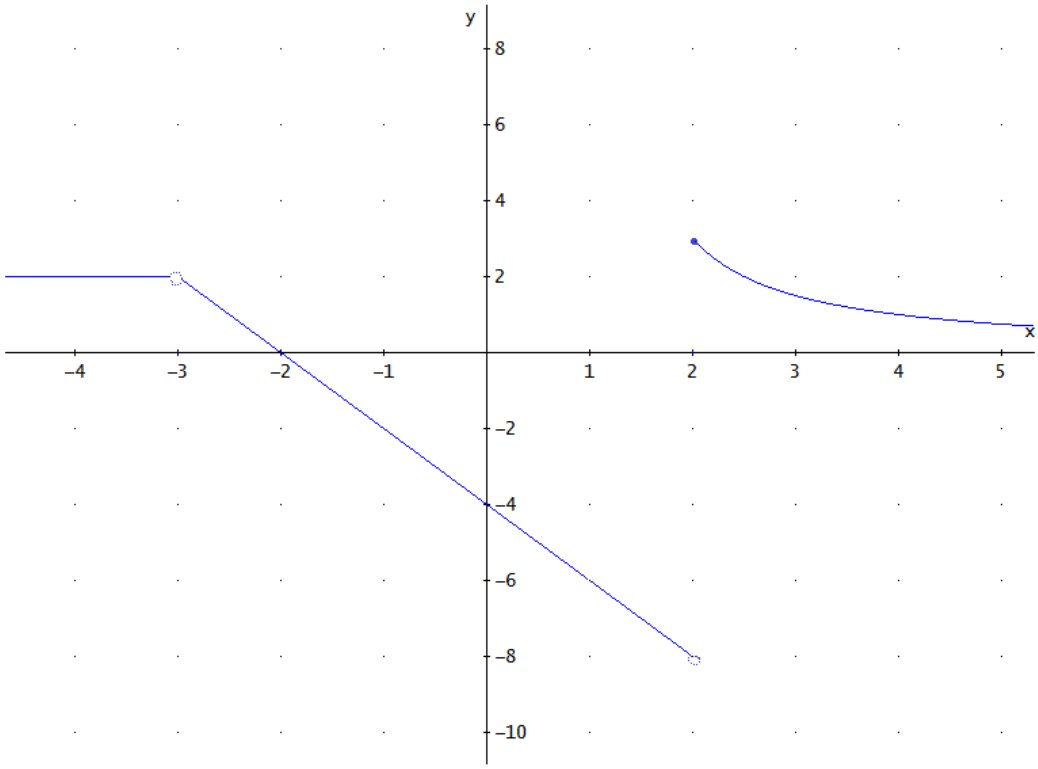
Equation: $y-6=\frac{5}{2}(x-6)\rightarrow y-6=\frac{5}{2}x-15\rightarrow y=\frac{5}{2}x-9$

f) Perpendicular bisector of AC: point $M\left(\frac{5}{2},\frac{9}{2}\right)$, perpendicular to AC, gradient of

$AC\rightarrow m_{AC}=\frac{y_2-y_1}{x_2-x_1}=\frac{6-3}{6+1}=\frac{3}{7}$; perpendicular $m'=-\frac{7}{3}$

Equation: $y-\frac{9}{2}=-\frac{7}{3}\left(x-\frac{5}{2}\right)\rightarrow y-\frac{9}{2}=-\frac{7}{3}x+\frac{35}{6}\rightarrow y=-\frac{7}{3}x+\frac{31}{3}$

5. $f(x)=\begin{cases} 2 & x<-3\rightarrow \text{horizontal line} \\ -2x-4 & -3<x<2\rightarrow \text{line, slope } (-2), \text{ intercepts } -y\ (-4) \\ \frac{3}{x-1} & x\geq 2\rightarrow \text{hyperbola with AH } y=0 \text{ and AV } x=1 \end{cases}$



a) Domain and range. $\text{Dom}=\Re-\{-3\}$; $R=(-8,3]$

b) Increasing and decreasing intervals:

Constant in $(-\infty,-3)$ Decreasing in $(-3,2)\cup(2,+\infty)$

c) It has a removable discontinuity in x = -3 and a jump discountuity in x=2

It is continuous in $\Re-\{-3,2\}$

6. Find the equation of a circle that has a diameter with the endpoints given by the points A(-1 , 2) and B(3 , 2).

The centre of the circle is the midpoint of AB, and the radius is the distance between M and A (or B)

$M\left(\frac{-1+3}{2},\frac{2+2}{2}\right)=(1,2)$; $r=d(M,A)=\sqrt{(-1-1)^2+(2-2)^2}=\sqrt{4}=2$

Equation: $(x-1)^2+(y-2)^2=4$