1st TERM GENERAL EXAM

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a. $-\sqrt{7}$ **b.** 1.25

a. $\sqrt{4}$ **d.** $-0.8\widehat{3}$

2) Calculate and simplify:

a. $\sqrt{18a}-\sqrt{\frac{128a}{9}}+\sqrt{450a}-\sqrt{50a}=$ b. $\sqrt{1+\sqrt{6+\sqrt{5+\sqrt{16}}}}=$

3) Calculate and simplify:

a. $\frac{\sqrt{8}.\sqrt[3]{12}.\sqrt[6]{3}}{\sqrt{32}}=$ **b.** $\left(\sqrt[3]{\sqrt{\sqrt[4]{2^3}}}\right)^2=$

4) Rationalise and simplify:

a. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ b. $\frac{a}{2\sqrt[3]{a}}$

5) Solve: $x^4-x^3-13x^2+x+12=0$

6) Solve: $\sqrt{3x-2}-\sqrt{x+3}=-1$

7) Solve: $\frac{x-1}{x^2-x}-\frac{1}{x^2-1}=\frac{1}{2(x+1)}$

8) Solve: $(x^2-1)^2-(2x+1)(2x-1)=3(2-x^2)$

9) Solve the following simultaneous equation: $\left.\begin{array}{l}\frac{2}{x}+\frac{1}{y}=1\\ x-y=0\end{array}\right\}$

10) Solve the following simultaneous equation: $\left.\begin{array}{l}2x+y=6\\ \sqrt{x}-y=-3\end{array}\right\}$

SOLUTION

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a. $-\sqrt{7}$ Real, Irrational, negative

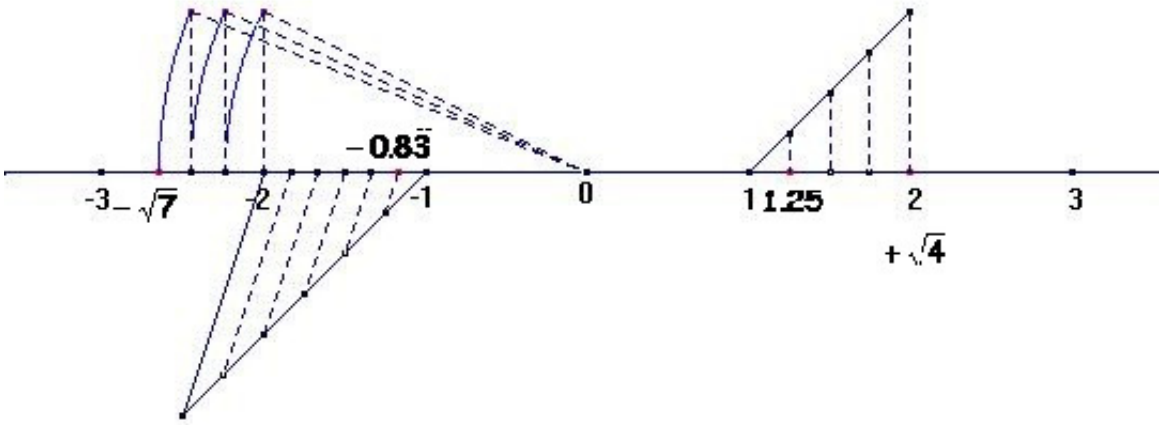
b. 1.25 Real, Rational

c. $+\sqrt{4}$ Real, Rational, Natural (+2)

d. $-0.8\widehat{3}$ Real, Rational, Recurrent decimal, negative

$$1.25=\frac{125}{100}=\frac{5}{4};\quad N=0.8333...\begin{cases}100N=83.333...\\10N=8.3333....\end{cases}\rightarrow 90N=75\rightarrow N=\frac{75}{90}=\frac{5}{6}$$

$$-0.8\widehat{3}=-\frac{5}{6}$$



2) Calculate and simplify:

a. $\sqrt{18a}-\sqrt{\frac{128a}{9}}+\sqrt{450a}-\sqrt{50a}=3\sqrt{2a}-\frac{8}{3}\sqrt{a}+15\sqrt{2a}-5\sqrt{2a}=$

$$=\left(3-\frac{8}{3}+15-5\right)\sqrt{2a}=\frac{9-8+45-15}{3}\sqrt{2a}=\frac{31}{3}\sqrt{2a}$$

b. $\sqrt{1+\sqrt{6+\sqrt{5+\sqrt{16}}}}=\sqrt{1+\sqrt{6+\sqrt{5+4}}}=\sqrt{1+\sqrt{6+3}}=\sqrt{1+3}=2$

3) Calculate and simplify:

a. $\frac{\sqrt{8}\cdot\sqrt[3]{12}\cdot\sqrt[6]{3}}{\sqrt{32}}=\frac{\sqrt[6]{(2^3)^3}\cdot\sqrt[6]{2^4\cdot 3^2}\cdot\sqrt[6]{3}}{\sqrt[6]{(2^5)^3}}=\frac{\sqrt[6]{2^9\cdot 2^4\cdot 3^2\cdot 3}}{\sqrt[6]{2^{15}}}=\sqrt[6]{\frac{2^{13}\cdot 3^3}{2^{15}}}=\sqrt[6]{\frac{3^3}{2^2}}$

b. $\left(\sqrt[3]{\sqrt{\sqrt[4]{2^3}}}\right)^2=\sqrt[24]{(2^3)^2}=\sqrt[24]{2^6}=\sqrt[4]{2}$

4) Rationalise and simplify:

a. $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\dfrac{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}=\dfrac{\sqrt{5^2}+2\sqrt{3}\sqrt{5}+\sqrt{3^2}}{\sqrt{5^2}-\sqrt{3^2}}=\dfrac{8+2\sqrt{15}}{2}=4+\sqrt{15}$

b. $\dfrac{a}{2\sqrt[3]{a}}=\dfrac{a\sqrt[3]{a^2}}{2\sqrt[3]{a}\sqrt[3]{a^2}}=\dfrac{a\sqrt[3]{a^2}}{2\sqrt[3]{a^3}}=\dfrac{a\sqrt[3]{a^2}}{2a}=\dfrac{\sqrt[3]{a^2}}{2}$

5) Solve: $x^4-x^3-13x^2+x+12=0$

Factors of 12: $\pm1,\pm2,\pm3,\pm6,\pm12$

$P(x)=x^4-x^3-13x^2+x+12$

$P(1)=1^4-1^3-13+1+12=0;\quad P(-1)=(-1)^4-(-1)^3-13-1+12=0$

	1	-1	- 13	+1	+12
1		+1	0	-13	-12
	1	0	- 13	-12	0
-1		-1	+1	+12	
	1	-1	- 12	0	

$x^2-x-12=0$

$x=\dfrac{1\pm\sqrt{1+48}}{2}=\begin{cases}4\\-3\end{cases}$

Solution: $x=1, x=-1, x=4, x=-3$

6) $\sqrt{3x-2}-\sqrt{x+3}=-1\rightarrow\sqrt{3x+2}=-1+\sqrt{x+3}$ to the power of two:

$3x-2=1-2\sqrt{x+3}+x+3\rightarrow2\sqrt{x+3}=-2x+6\rightarrow\left(2\sqrt{x+3}\right)^2=(6-2x)^2\rightarrow$

$4(x+3)=4x^2+36-24x\rightarrow4x^2-28x+24=0\rightarrow\begin{cases}x=6\\x=1\end{cases}$

Check: $\sqrt{3\cdot6-2}-\sqrt{6+3}=-1\rightarrow\sqrt{16}-\sqrt{9}=4-3\neq-1$ It is not a solution

$\sqrt{3\cdot1-2}-\sqrt{1+3}=-1\rightarrow\sqrt{1}-\sqrt{4}=1-2=-1$ It is a solution **Solution: x = 1**

7) $\dfrac{x-1}{x^2-x}-\dfrac{1}{x^2-1}=\dfrac{1}{2(x+1)}$

$x^2-x=x(x-1);$

$x^2-1=(x+1)(x-1)$ m.c.m.= $2x(x+1)(x-1)$

$2(x+1)=2(x+1)$

$$\frac{x-1}{x^2-x}-\frac{1}{x^2-1}=\frac{1}{2(x+1)}\rightarrow\frac{(x-1)2(x+1)-2x}{2x(x+1)(x-1)}=\frac{x(x-1)}{2x(x+1)(x-1)}\rightarrow$$

$$2x^2-2-2x=x^2-x\rightarrow x^2-x-2=0\rightarrow\begin{cases}x=2\\x=-1\end{cases}$$

x= -1 It is not a solution (denominator 0) **Solution: x = 2**

8) Solve: $(x^2-1)^2-(2x+1)(2x-1)=3(2-x^2)$

$$x^4-2x^2+1-(4x^2-1)=6-3x^2\rightarrow x^4-2x^2+1-4x^2+1=6-3x^2$$

$$x^4-3x^2-4=0\rightarrow\text{biquadratic}\rightarrow z=x^2\Rightarrow z^2-3z-4=0$$

$$z=\frac{3\pm\sqrt{9+16}}{2}=\begin{cases}4\\-1\end{cases}\rightarrow\begin{cases}x=\pm\sqrt{4}\\x=\pm\sqrt{-1}\end{cases}\rightarrow x=\pm2$$

Solution: x = 2, x = -2

9) Solve the following simultaneous equation: $\left.\begin{matrix}\frac{2}{x}+\frac{1}{y}=1\\x-y=0\end{matrix}\right\}$

$$\left.\begin{matrix}\frac{2y}{xy}+\frac{x}{xy}=\frac{xy}{xy}\\x-y=0\end{matrix}\right\}\rightarrow\left.\begin{matrix}2y+x=xy\\x=y\end{matrix}\right\}\rightarrow 2y+y=y^2\rightarrow y^2-3y=0\rightarrow y(y-3)=0$$

$$\rightarrow\begin{cases}y=0\text{ It is not a solution (division by zero)}\\y=3\rightarrow x=3\end{cases}$$

Solution: x = 3, y = 3

10) Solve the following simultaneous equation: $\left.\begin{matrix}2x+y=6\\\sqrt{x}-y=-3\end{matrix}\right\}$

$$\left.\begin{matrix}2x+y=6\\\sqrt{x}-y=-3\end{matrix}\right\}\rightarrow\left.\begin{matrix}y=6-2x\\y=\sqrt{x}+3\end{matrix}\right\}\Rightarrow 6-2x=\sqrt{x}+3\Rightarrow 3-2x=\sqrt{x}$$

$$(3-2x)^2=\left(\sqrt{x}\right)^2\Rightarrow 9-12x+4x^2=x\Rightarrow 4x^2-13x+9=0\rightarrow x=\begin{cases}\frac{9}{4}\\1\end{cases}$$

Check: $3-2x=\sqrt{x}\rightarrow\begin{cases}3-2\cdot\frac{9}{4}=\sqrt{\frac{9}{4}}\rightarrow 3-\frac{9}{2}\neq\frac{3}{2}\quad\text{No}\\3-2\cdot1=\sqrt{1}\rightarrow 3-2=1\quad\text{Yes}\rightarrow y=6-2\cdot1=4\end{cases}$

Solution: x = 1, y = 4