IES MURILLO

Maths 4th ESO

2nd TERM GENERAL EXAM

Name:_____

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1. Solve the equation: $\sqrt{x}+\sqrt{x-2}=2$

2. Solve the equation: $\frac{1}{x+2}-\frac{x+2}{x}=-\frac{7}{4}$

3. Solve by substitution and graphically: $\left.\begin{array}{l}2x-y-6=0\\ x(x-6)=6+y\end{array}\right\}$

4. Solve the system of inequalities: $\left.\begin{array}{l}\frac{x+y}{2}<x-1\\ \frac{x-y}{2}\geq y+1\end{array}\right\}$

5. Sketch the graph of the compound function:

$$f(x)=\begin{cases}3-x & x<-1\\ 4 & -1<x<1\\ 2^{x-1} & x\geq 1\end{cases}$$

   a) Domain and range

   b) Continuity

6. Calculate x in the following equations:

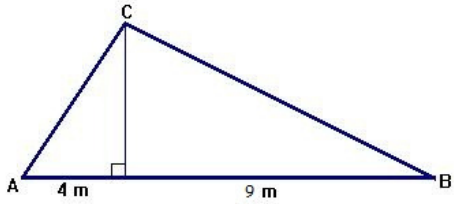
   a) $2^{1-x^2}=\frac{1}{256}$ b) $log_{25}(x-2)=\frac{1}{2}$

7. The time for a car to travel between two cities is inversely proportional to the rate of travel. If it takes 8 hours to travel from San Francisco to Los Angeles at a rate of 60 mph, how long would it take traveling at 75 mph?

   a) Analyse and describe the type of relation between speed and time.

   b) Graph the function.

8. Find the area and perimeter of the right triangle ABC (Don't use trigonometry!)



9. Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 46° and 65° respectively. If the height of the building is known to be h =75 m, find the distance between the two men.

10. One leg of a right triangle is seven centimetres shorter than the other leg. If the hypotenuse is 13 cm, find the length of the shorter leg.

SOLUTION

1. Solve the equation: $\sqrt{x}+\sqrt{x-2}=2$

$$\sqrt{x}+\sqrt{x-2}=2 \Rightarrow \sqrt{x-2}=2-\sqrt{x} \Rightarrow \left(\sqrt{x-2}\right)^2=\left(2-\sqrt{x}\right)^2$$

$$x-2=4-4\sqrt{x}+x \Rightarrow 4\sqrt{x}=6 \Rightarrow \sqrt{x}=\frac{3}{2} \Rightarrow x=\left(\frac{3}{2}\right)^2 \Rightarrow x=\frac{9}{4}$$

Checking: $\sqrt{\frac{9}{4}}+\sqrt{\frac{9}{4}-2}=\frac{3}{2}+\sqrt{\frac{1}{4}}=\frac{3}{2}+\frac{1}{2}=\frac{4}{2}=2$ Yes

2. Solve the equation: $\frac{1}{x+2}-\frac{x+2}{x}=-\frac{7}{4}$

$$\frac{4x}{4x(x+2)}-\frac{4(x+2)^2}{4x(x+2)}=-\frac{7x(x+2)}{4x(x+2)} \Rightarrow 4x-4\left(x^2+4x+4\right)=-7\left(x^2+2x\right)$$

$$4x-4x^2-16x-16=-7x^2-14x \Rightarrow 3x^2+2x-16=0$$

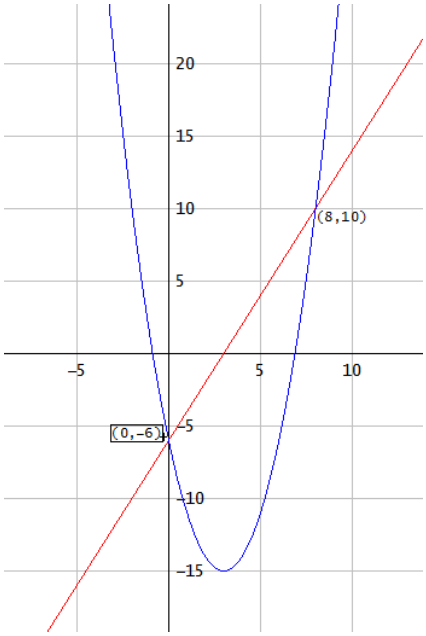
$$x=\frac{-2\pm\sqrt{4+192}}{6}=\frac{-2\pm 14}{6}=\begin{cases}2\\ -\frac{8}{3}\end{cases}$$

3. Solve by substitution and graphically: $\left.\begin{array}{l}2x-y-6=0\\ x(x-6)=6+y\end{array}\right\}$

$$\left.\begin{array}{l}2x-6=y\\ x^2-6x=6+y\end{array}\right\} \Rightarrow x^2-6x=6+2x-6 \Rightarrow x^2-8x=0 \Rightarrow \begin{cases}x=0\\ x=8\end{cases}$$

Solution: $\begin{cases}x=0 \rightarrow y=-6\\ x=8 \rightarrow y=10\end{cases}$

Graphically:



$\begin{cases}y=2x-6 & \rightarrow \text{straight line}\\ y=x^2-6x-6 & \rightarrow \text{parabola} \cup\end{cases}$

Vertex: $x=-\frac{-6}{2}=3 \Rightarrow (3,-15)$

Intercepts-y: $x=0 \rightarrow y=-6$

Intercepts-x: $x^2-6x-6=0$

$$x=\frac{6\pm\sqrt{36+24}}{2}=\frac{6\pm\sqrt{60}}{2}=\begin{cases}6.87\\ -0.87\end{cases}$$

Solution: $(0,-6)$ and $(8,10)$

4. Solve the system of inequalities: $\left.\begin{array}{l}\frac{x+y}{2} < x-1 \\ \frac{x-y}{2} \geq y+1\end{array}\right\}$ we solve it graphically:

$$\left.\begin{array}{l}\frac{x+y}{2} = x-1 \\ \frac{x-y}{2} = y+1\end{array}\right\} \rightarrow \left.\begin{array}{l}x+y = 2x-2 \\ x-y = 2y+2\end{array}\right\} \rightarrow \left.\begin{array}{l}y = x-2 \\ y = \frac{x-2}{3}\end{array}\right\}$$ We sketch both lines:

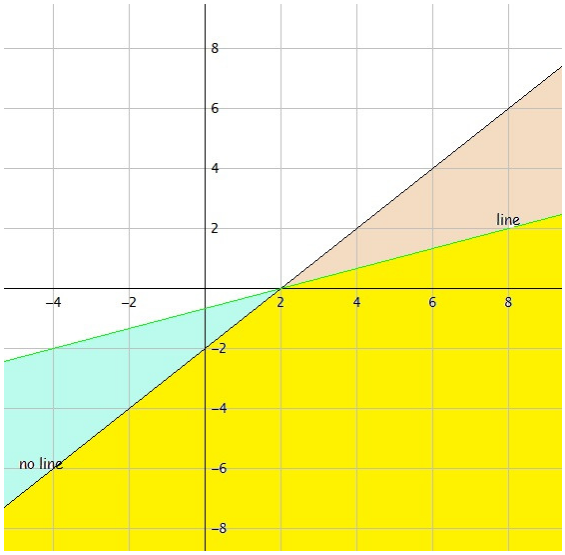
$\frac{x+y}{2} < x-1$ check (0,0)

$\frac{0}{2} < 0-1 \rightarrow 0 < -1$ NO

$\frac{x-y}{2} \geq y+1$ check (0,0)

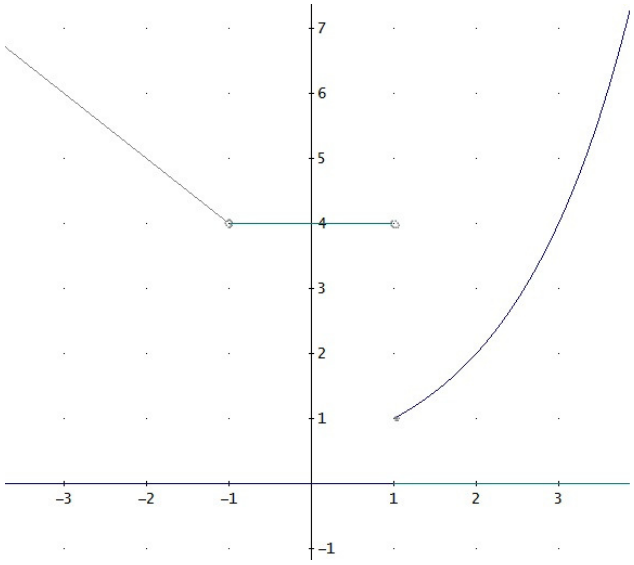
$\frac{0}{2} \geq 0+1 \rightarrow 0 \geq 1$ NO

the solution is the region that **both** inequalities cover (YELLOW)



5. Sketch the graph of the compound function:

$$f(x) = \begin{cases} 3-x & x < -1 \rightarrow \text{straight line} \\ 4 & -1 < x < 1 \rightarrow \text{horizontal line} \\ 2^{x-1} & x \geq 1 \rightarrow \text{exponential, 1 to the right} \end{cases}$$



a)Domain and range

$\text{Dom} = R - \{1\}$

$\text{Range} = [1, +\infty)$

b)Continuity

It has a removable discontinuity in -1 and a jump discontinuity in 1.

6. Calculate x in the following equations:

a) $2^{1-x^2} = \frac{1}{256} \Rightarrow 2^{1-x^2} = 2^{-8} \Rightarrow 1-x^2 = -8 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

b) $log_{25}(x-2) = \frac{1}{2} \Rightarrow 25^{\frac{1}{2}} = x-2 \Rightarrow \sqrt{25} = x-2 \Rightarrow 5 = x-2 \Rightarrow x = 7$

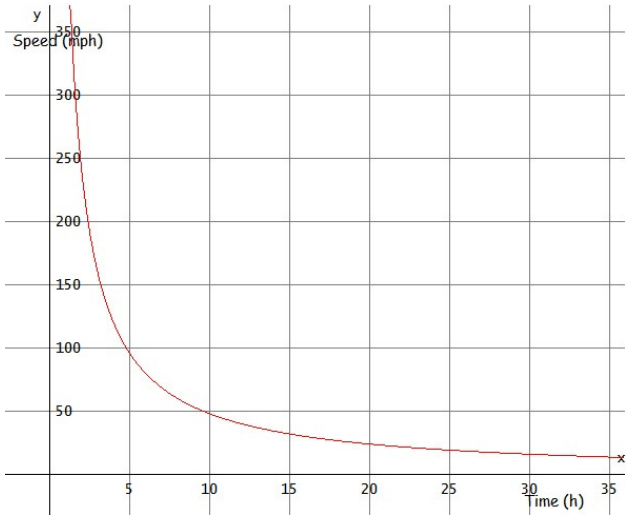
7. The time for a car to travel between two cities is inversely proportional to the rate of travel. If it takes 8 hours to travel from San Francisco to Los Angeles at a rate of 60 mph, how long would it take traveling at 75 mph?

$8 \cdot 60 = 480$ miles from San Francisco to Los Angeles

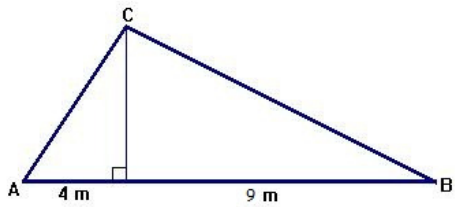
$480 \div 75 = 6.4$ 6 hours 24 minutes

a) Analyse and describe the type of relation between speed and time. time $= 480 \div$ velocity , so it is an inversely proportional function, with formula $y = \frac{480}{x}$ (hyperbola)

b) Graph the function



8. Find the area and perimeter of the right triangle ABC (Don't use trigonometry!)



Theorem of the height:

$h^2 = 4 \cdot 9 = 36 \Rightarrow h = \sqrt{36} = 6$ m

Theorem of the legs:

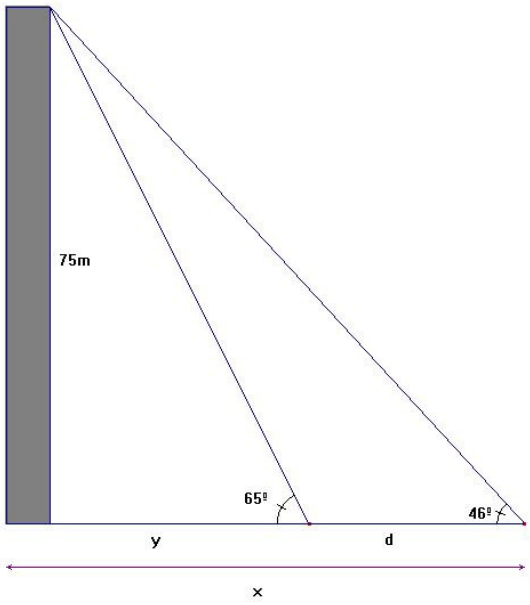
$b^2 = 4 \cdot 13 = 52 \Rightarrow b = \sqrt{52} = 7.21$m

Pythagorean Theorem:

$13^2 = 52 + c^2 \Rightarrow c = \sqrt{169-52} = 10.82$m

Area: $A = \frac{13 \cdot 6}{2} = 39\ m^2$ Perimeter: $P = 13 + 7.21 + 10.82 = 31.03$ m

9. Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 46° and 65° respectively. If the height of the building is known to be *h* =75 m, find the distance between the two men.



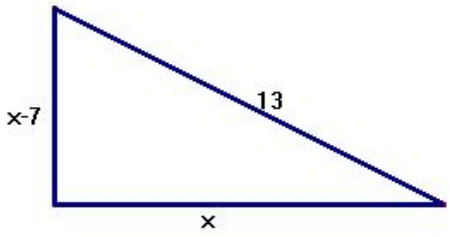
$$\left.\begin{array}{l} tan\,65 = \dfrac{75}{y} \\ tan\,46 = \dfrac{75}{x} \end{array}\right\} \Rightarrow \left.\begin{array}{l} y = \dfrac{75}{tan\,65} \\ x = \dfrac{75}{tan\,46} \end{array}\right\}$$

$$\left.\begin{array}{l} y = \dfrac{75}{tan\,65} = 34.97 \\ x = \dfrac{75}{tan\,46} = 72.43 \end{array}\right\} d = x - y$$

d=72.43 – 34.97 = 37.46 m

The distance between the two men is 37.46 m

10. One leg of a right triangle is seven centimetres shorter than the other leg. If the hypotenuse is 13 cm, find the length of the shorter leg.



Pythagorean Theorem:

$$13^2 = (x-7)^2 + x^2 \Rightarrow 169 = x^2 - 14x + 49 + x^2$$

$$2x^2 - 14x - 120 = 0 \rightarrow x^2 - 7x - 60 = 0$$

$$x = \frac{7 \pm \sqrt{49+240}}{2} = \frac{7 \pm \sqrt{289}}{2} = \begin{cases} 12 \\ -5 \end{cases}$$

The shorter leg is 12 – 7 = 5 cm long