IES MURILLO

Maths 4th ESO

2nd TERM GENERAL EXAM

Name:_____

Remember: in each question, write the steps you have taken to reach the solution.

1. Solve the following equations: (1.5 points)

a) $\sqrt{3x+4}+2x-4=0$ b) $\frac{x+5}{x-5}-\frac{5-x}{x+5}=\frac{10}{3}$

2. Solve the system of inequalities: $\left.\begin{array}{l} x+y\geq 9 \\ -2x+3y>12 \end{array}\right\}$ (1 point)

3. Solve the inequality: $\frac{x^2-4x-5}{x-1}\geq 0$ (1 point)

4. Sketch the graph of the compound function: (1.5 points)

$$f(x)=\begin{cases} 2-x & \text{if} \quad x<-1 \\ 3 & \text{if} \quad -1<x<2 \\ -\sqrt{x+2} & \text{if} \quad x\geq 2 \end{cases}$$

a) Domain and range
b) Increasing and decreasing intervals
c) Continuity

5. Calculate x in the following equations: (1.5 points)

a) $2^{x+3}=\frac{1}{16}$ b) $\log_3(x-2)=2$ c) $\log_x \frac{1}{4}=-2$

6. Solve algebraically and graphically the simultaneous equation:

$\left.\begin{array}{l} 2+y=x \\ y=\frac{2}{x-3}+2 \end{array}\right\}$ (Don't use a table data) (2 points)

7. A boy stands at a top of 135 m high building and throw a stone upward. The stone travel upward for a while then eventually is pulled by gravity down to the ground. The height of the stone above the ground is given by the function $f(t)=-16t^2+36t+135$ (t time in seconds)

a) Draw the graph (don't use a table data).
b) How high does the ball rise before started to drop downward?
c) After how many seconds does the ball hit the ground? (1.5 points)

SOLUTION

1. Solve the following equations:

a) $\sqrt{3x+4}+2x-4=0 \rightarrow \sqrt{3x+4}=4-2x \rightarrow \left(\sqrt{3x+4}\right)^2=(4-2x)^2$

$$3x+4=16-16x+4x^2 \Rightarrow 4x^2-19x+12=0 \Rightarrow x=\frac{19\pm 13}{8}=\begin{cases}4\\ \frac{3}{4}\end{cases}$$

$\sqrt{3\cdot 4+4}=4-2\cdot 4 \rightarrow 4\neq -4$ No

Checking: $\sqrt{3\cdot\frac{3}{4}+4}=4-2\cdot\frac{3}{4} \rightarrow \sqrt{\frac{9}{4}+\frac{16}{4}}=4-\frac{3}{2} \rightarrow \frac{5}{2}=\frac{5}{2}$ Yes

Solution: $x=\frac{3}{4}$

b) $\frac{x+5}{x-5}-\frac{5-x}{x+5}=\frac{10}{3} \rightarrow \frac{3(x+5)(x+5)}{3(x+5)(x-5)}-\frac{3(5-x)(x-5)}{3(x+5)(x-5)}=\frac{10(x-5)(x+5)}{3(x+5)(x-5)}$

$3(x^2+10x+25)-3(-x^2+10x-25)=10(x^2-25)$

$3x^2+30x+75+3x^2-30x+75=10x^2-250$

$-4x^2=-400 \Rightarrow x^2=100 \Rightarrow x=\pm 10$

2. Solve the system of inequalities: $\left.\begin{array}{l}x+y\geq 9\\ -2x+3y>12\end{array}\right\}$

Graphically: $\left.\begin{array}{l}x+y=9\\ -2x+3y=12\end{array}\right\} \rightarrow \left.\begin{array}{l}y=9-x\\ y=\frac{12+2x}{3}\end{array}\right\}$ We sketch both lines

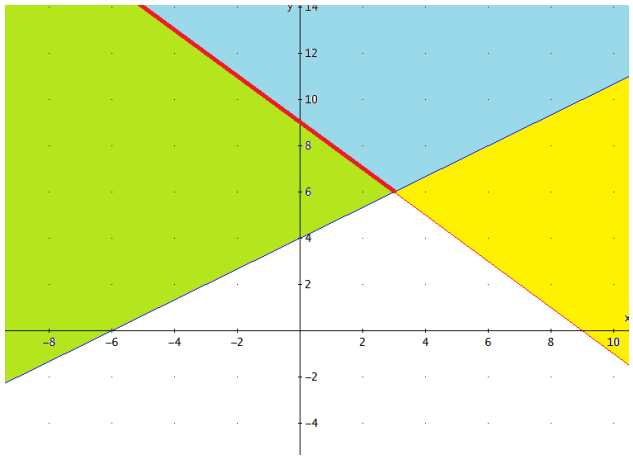
$x+y\geq 9$ check (0,0)

$0+0\geq 9 \rightarrow 0\geq 9$ NO

$-2x+3y>12$ check (0,0)

$0+0>12 \rightarrow 0>12$ NO

The solution region is the one blue (the region that both inequalities cover), with the red line



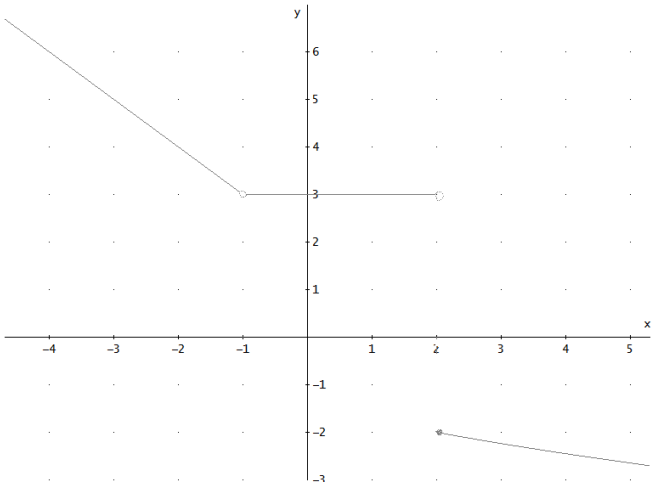
3. Solve the inequality: $\frac{x^2-4x-5}{x-1}\geq 0 \rightarrow x^2-4x-5=0 \Rightarrow x=\frac{4\pm\sqrt{36}}{2}=\begin{cases}5\\ -1\end{cases}$

$\frac{(x-5)(x+1)}{x-1}\geq 0$ we are going to study the sign:

Solution: $[-1,1)\cup[5,+\infty)$

4. Sketch the graph of the compound function:

$$f(x)=\begin{cases} 2-x & \text{if } x<-1 \rightarrow \text{straight line, slope } -1, \text{intercepts - y } 2 \\ 3 & \text{if } -1<x<2 \rightarrow \text{horizontal line} \\ -\sqrt{x+2} & \text{if } x\geq 2 \rightarrow \text{radical , Domain } [-2,+\infty) \end{cases}$$



a)Domain and range

$\text{Dom} = R-\{-1\}$

$\text{Range} = (-\infty,-2]\cup[3,+\infty)$

b) Decreasing in $(-\infty,-1)\cup(2,+\infty)$

Constant in $(-1,1)$

c) Continuity

It has a removable discontinuity in -1 and a jump discontinuity in 2.

5. Calculate x in the following equations:

a) $2^{x+3}=\frac{1}{16}\rightarrow 2^{x+3}=2^{-4}\rightarrow x+3=-4\rightarrow x=-7$

b) $\log_3(x-2)=2\rightarrow 3^2=x-2\rightarrow 9=x-2\rightarrow x=11$

c) $\log_x\frac{1}{4}=-2\rightarrow x^{-2}=\frac{1}{4}\rightarrow x^{-2}=2^{-2}\rightarrow x=2$

6. Solve algebraically and graphically the simultaneous equation:

$$\left.\begin{array}{l} 2+y=x \\ y=\dfrac{2}{x-3}+2 \end{array}\right\}$$ Graphically, straight line and hyperbola

Line: $y=x-2$, slope +1, intercepts-y -2

Hyperbola: Vertical asymptote x = 3, Horizontal asymptote y =2

Algebraically:

$$\left.\begin{array}{l} 2+y=x \\ y=\dfrac{2}{x-3}+2 \end{array}\right\} \rightarrow \left.\begin{array}{l} y=x-2 \\ y=\dfrac{2}{x-3}+2 \end{array}\right\} \rightarrow x-2=\frac{2}{x-3}+2 \rightarrow (x-2)(x-3)=2+2(x-3)$$

$$x^2-3x-2x+6=2+2x-6 \rightarrow x^2-7x+10=0 \rightarrow x=\frac{7\pm\sqrt{9}}{2}=\begin{cases}5 \rightarrow y=3\\ 2 \rightarrow y=0\end{cases}$$

Solution: they intercept in (5,3) and (2,0)

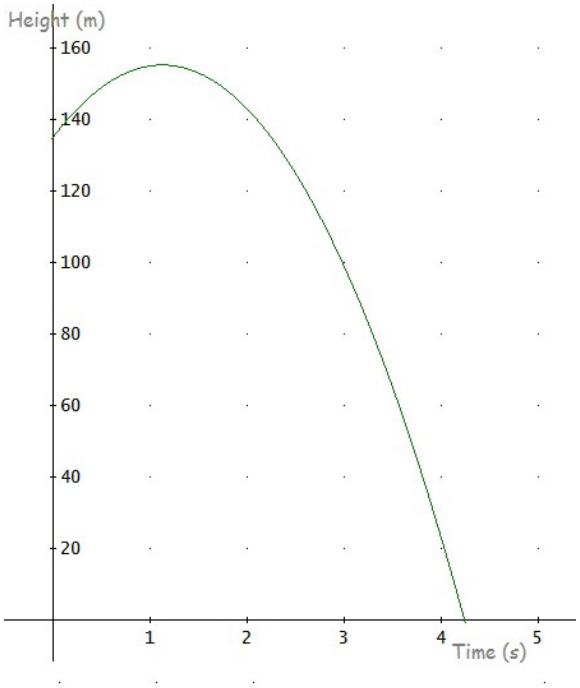
7. A boy stands at a top of 135 m high building and throw a stone upward. The stone travel upward for a while then eventually is pulled by gravity down to the ground. The height of the stone above the ground is given by the function $f(t)=-16t^2+36t+135$ (t time in seconds)

a) Draw the graph (don't use a table data). $y=-16x^2+36x+135 \rightarrow \cap$

Vertex $x=-\frac{36}{-32}=\frac{9}{8} \rightarrow y=-16\cdot\left(\frac{9}{8}\right)^2+40\cdot\frac{9}{8}+135=155.25 \qquad V\left(\frac{9}{4},144\right)$

Intercepts- y $y=135$

Intercepts-x $-16x^2+36x+135=0 \rightarrow x=\frac{-36\pm\sqrt{9936}}{-32}=\begin{cases}4.24\\ -1.99\end{cases}$



b) How high does the ball rise before started to drop downward? 155.25 metres

c) After how many seconds does the ball hit the ground? After 4.24 seconds